MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Dr. J. L. E. DREYER, PRESIDENT, in the Chair.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Harihara Subramani Aiyar, M.A., 118 Belsize Road, London, N.W. 6 (proposed by L. N. G. Filon);

Rev. William Benjamin Hine, M.A., Chaplain and Science Master, Royal Savoy School, London, S.E. 22 (proposed by J. M. F. Dumphreys);

Herbert Horrocks, 1 Westminster Road, Chorley, Lancs (proposed by A. S. Eddington);

Rev. C. B. Johnson, 134 Lorne Terrace, Preston New Road, Blackburn, Lancs (proposed by A. L. Cortie);

Miss Katharine Pember, The Warden's Lodgings, All Souls College, Oxford (proposed by L. N. G. Filon);

George A. Taylor, A.M.I.E. of Australia, 20 Loftus Street, Sydney, N.S.W., Australia (proposed by W. E. Cooke); and

Miss Elizabeth Williamson, Hill Hall, Theydon Mount, Epping, Essex (proposed by L. N. G. Filon).

Eighty presents were announced as having been received since the last meeting, including, amongst others:—

Berlin, Astronomisches Rechen-Institut Veröffentlichungen, No. 44.

G. E. Hale, The Depths of the Universe.

Kasan, Observatoire Engelhardt, Publications Nos. 8, 9, 10.

The Address was delivered by the President, after which the Gold Medal was handed to Professor A. S. Eddington, to whom it had been awarded for his work on Star-streaming, on the Internal Constitution of a Star, and on Generalised Relativity.

ADDRESS

Delivered by the President, Dr. J. L. E. Dreyer, on presenting the Gold Medal of the Society to Professor A. S. Eddington, F.R.S.

It was announced at the January meeting that the Council had considered the question of the most suitable date for the presentation of the medal. It was thought that as the award is only made in January, the Medallist would more frequently be able to attend in person to receive the medal if the presentation were delayed till later in the session, in general till the June meeting.

I shall now have the pleasure of laying before the Society the grounds for the award by the Council of the Gold Medal to Professor Eddington for his researches on Star-Streaming, on the Internal Constitution of a Star, and on Generalised Relativity. These three subjects, in the order here named, embrace practically the whole of the activity of our Medallist.

After a distinguished career at Cambridge, Mr. Eddington was appointed a Chief Assistant at the Royal Observatory, Greenwich, in 1906, where he remained till 1913, when he returned to Cambridge as Plumian Professor. The very first paper which he communicated to our Society, in 1906 November, dealt with the first of the three subjects I have mentioned, being entitled "The Systematic Motions of the Stars." In 1904 September Kapteyn had made the startling announcement that his examination of the proper motions of Bradley's stars had convinced him that the true motions of a large number of stars, allowance being made for the solar motion, are not at random, but that there are two "favoured" directions of motion. Kapteyn suggested that there are two systems or "drifts" of stars. He repeated this announcement in 1905 at the meeting of the British Association in South Africa, but it did not create much sensation at the time. The importance and the "revolutionary character" of Kapteyn's discovery seemed, however, to Mr. Eddington to make it very desirable to test it by an independent investigation, and for this purpose he made use of the proper motions of the Groombridge Catalogue, which had a short time before been determined by Dyson and Thackeray. While the Bradley stars are all bright stars, the Groombridge Catalogue includes a large proportion of stars between the 7th and the 9th magnitudes, and it was very important to find out whether these fall into Kapteyn's two drifts. In his first paper Mr. Eddington discussed the subject in all its bearings, dealing with most of the phenomena connected with the twofold star-drift, and at once raised Kapteyn's announcement to the rank of an important discovery. This paper was in the course of the next eight or nine years followed by a number of others on the construction of the Heavens, all published in the *Monthly Notices*. The subject naturally also attracted the attention of other investigators, such as Dyson, Hough, Halm, and Schwarzschild, and in the beginning of

1914 Professor Eddington brought together the results of all this work, his own and that of others, in a very valuable book entitled *Stellar Movements and the Structure of the Universe.*

The first question to be settled was, whether the stars really tend to move in two favoured directions. A tendency to move in *one* direction would disappear when the parallactic motion (that is, the apparent displacement of a star caused by the motion of the sun) is removed. But a tendency in two directions, if found, can only be an intrinsic property of the individual motions of the stars. To exhibit this phenomenon, Mr. Eddington adopted a novel plan. Taking a limited region of the sky, in which the apparent motions are seen projected on what is practically a plane, he counted the number of stars observed to be moving in different directions. Proceeding by steps of 10°, tables were formed of the number of stars moving towards position angles 0°, 10°, 20° . . . 350°. The results were shown on polar diagrams, the curves being drawn so that the radius is proportional to the number of stars moving in the corresponding direction. But first it was investigated what form of curve would be obtained if the hypothesis of random motion were correct, and it was shown that this form varies with the ratio of the velocity of the parallactic motion to the mean peculiar velocity. It is, of course, symmetrical with regard to the direction of this motion, and it does not in any way depend on the distance of the stars. When these single-drift curves were compared with the curves representing the observed distribution of proper motions in different parts of the sky, it was seen at once that none of the theoretical curves could be made to fit any of the curves derived from observations. There were two favoured directions in which the stars were streaming, and it was therefore necessary to analyse each region separately to see if the observed curve could be accounted for on the assumption that it is compounded of two single-drift curves. This was found to be distinctly the case. Treating each drift separately and plotting the directions of the drifts on a globe as great circles, these (produced backwards) were found for Drift I to pass close to the point R.A. 270°, Decl. +18°. For Drift II. the same accuracy of convergence was not to be expected, as the curves were not very elongated, and it was not possible to tell with precision which way they were pointing. The best position for the apex of Drift II. was found to be 112° + 58°.

Encouraged by these results, Mr. Eddington in 1907 December read his next paper "On the Mean Distances of the Groombridge Stars," and in the following June another "On the Mathematical Theory of Two-star Drifts and on the Systematic Motions of Zodiacal Stars." But his most exhaustive investigation was entitled "The Systematic Motions of the Stars of Professor Boss's Preliminary General Catalogue" (*M.N.*, 1910 Nov.). This catalogue gives the proper motions of 6188 stars, well distributed over the whole sky and all well observed, so that the proper motions have a high degree of accuracy, all the brighter stars being included and the fainter ones being fairly representative and not selected on account of the amount of their proper motion. Practically all the important results of Mr. Eddington's researches are accordingly based on his analysis of this catalogue.

After excluding certain classes of stars (those of the Orion type,* members of moving clusters, and the fainter components of binary systems) there remained 5322 stars for investigation. These were divided between seventeen regions of the sky, each region consisting of an area in the northern hemisphere, together with an antipodal area in the southern hemisphere. The principal object of this device was to secure a sufficient number of stars in each region without unduly extending the area that must be treated as plane. There is the further advantage that any error in the adopted precession will affect the two portions of one region in exactly opposite directions, so that its effects will be eliminated to a great extent. I need not exhibit the diagrams showing the proper motions; they are given in vol. lxxi. of the *Monthly Notices*, and similar ones have recently been shown on the screen by Dr. Smart when extending Professor Eddington's investigation to faint stars. The diagrams and the seventeen tables of the number of stars moving within each 10° of position angle give very strong evidence for the existence of two star-streams. On each diagram an arrow points to the antapex of the solar motion (R.A. 90°·5, Decl. − 34·3), while two arrows marked I. and II. point to the apices of the two drifts found from the collected results of this discussion. The tendency to move in the directions of these two arrows is plainly seen, while there is no resemblance whatever to a symmetrical single-drift curve pointing along the antapex arrow. Proceeding as in his first paper, Mr. Eddington found that for Drift I. the convergence of the directions was very evident; but owing to the smaller velocity of Drift II., its direction in any region cannot be determined so accurately, and a greater deviation of the great circles was therefore to be expected. The analysis of the regions gives not only the directions but also their speeds in terms of a certain unit. The final results are—

		Drift I.	Drift II.
Apex	R.A.	90°·8	287°·8
	Decl.	− 14·6	− 64·1
	Speed	1·52	0·86

The result of this analysis of the Boss proper motions shows that the motions can be closely represented if there are two drifts. The first one contains about 60 per cent. of the stars and the second one 40 per cent. Their directions are inclined at an angle of 100°. These motions are measured relative to the sun. The points in the sky towards which the motion of one drift relative to the other is directed are called the vertices. Their positions are—

R.A.	94°·2	274°·2
Decl.	+ 11·9	− 11·9

while the solar apex was found to be 267°·3 + 36°·4, in good agreement with Boss's determination from the same catalogue by the ordinary

* These were omitted on account of their tendency to form vague, moving clusters. It was afterwards found that they do not participate in the drift motions.

method, 270°·5 + 34°·3. The vertices fall exactly in the galactic plane, so that the relative motion of the two drifts is parallel to that plane.

A valuable confirmation of this phenomenon has been afforded by the detection of it in the spectroscopically measured velocities in the line of sight. The discussion of this subject had first been attempted by Hough and Halm, and next (for Type A only) by Campbell, when Professor Eddington, jointly with the late W. E. Hartley, investigated it, making use of the radial velocities published by Campbell. The main result was that there is a close agreement between the results derived from the proper motions and those based on the radial velocities. The vertex was found from 1086 stars of types A to M to be in R.A. 94°·6, Decl. + 12°·5. There was no evidence of a progressive change with spectral type in the position of the vertex.

When the predominance of the two opposite directions of star-streaming was definitively established, several important questions at once demanded answers. One of these, whether the stars of the two streams are on an average of the same magnitude and spectral type, was discussed in the very first paper of our Medallist. It turned out that there is no important difference in the magnitudes of the stars constituting the two drifts, though it should perhaps not be ignored that the discussions of the faint stars, such as the Groombridge or Carrington stars, have given a higher proportion belonging to Drift II. than the discussions of brighter stars, such as those of Bradley or Boss. As regards the spectral types, the result was more ambiguous, and all that could be said was that stars of Types I. and II. are found in both streams, but that there is a somewhat higher proportion of Type II. among stars moving in the direction of Drift II. than of Drift I.

Of even greater interest is the question whether the two streams are actually intermingled in space. A very great majority of the stars which are bright enough to be dealt with in these investigations are apparently grouped in the form of a lens-shaped system, not far from the centre of which our sun is situated, the median plane of the lens being the same as the mean plane of the star clouds of the Milky Way. It might very well be, that one of the streams consists of a cluster of stars surrounding the sun, moving relatively to a background of stars constituting the other stream. The absence of any appreciable correlation between magnitude and drift makes such an explanation very improbable, as it would be expected that the stars of the background would be fainter on the average than those of the nearer swarm. The question was, however, treated more definitely by using the amounts of the proper motions to estimate the distances of the drifts. Everywhere the two drifts were found to be at nearly the same mean distance, and the more distant stars belong to both drifts. On the whole, the mixture is remarkably complete.

The two-drift theory has furnished explanations of more than one fact which formerly seemed very puzzling. In earlier determinations of the solar apex it was always found that the declination of the apex decreased as the amount of the proper motions increased. This is now seen to be due to the much greater velocity of Drift I. relative to the sun, owing to which the stars belonging to it have on an average greater

proper motions than the stars of Drift II. Thus, the larger the motions discussed, the greater will be the proportion of Drift I. stars, and the nearer will be the resulting apex to the apex of Drift I. Similarly, the reason why stars of G, K, M types give a higher declination of the solar apex may be due to the increased proportion of Drift II. in these types.

In his first paper Mr. Eddington suspected, and in his investigation of Boss's catalogue he clearly showed, that when comparing the observed distribution of velocities with that calculated on the two-drift theory there is always some excess of stars moving towards the solar antapex. This he attributed to a third small stream of much less importance than the two great drifts. This was examined in detail by Halm, who showed that there is a third drift, Drift O, which is practically at rest in space. This is a well-known feature of the stars of the Orion or B type, shown both by their very small radial velocities and by their proper motions. What there is of systematic motion seems to be purely parallactic, due to the motion of the sun through space. But this peculiarity of being at rest is also to some extent shared by stars not belonging to Type B. The latter have a tendency to aggregate into moving clusters, though the motion is generally very small. The great cluster in Scorpius and Centaurus (found simultaneously by Kapteyn and Eddington), the constellation of Orion, the Pleiades, and the Perseus cluster found by Eddington contain between them a considerable proportion of the stars of this type, the remainder having generally considerably smaller proper motions than the cluster stars.

The existence of these moving clusters the members of which, though often widely separated in space, must have preserved equal and parallel motions for immense periods of time, led our Medallist to investigate the dynamics of a globular stellar system. The result was that the stars move in undisturbed paths under the general attraction of the whole stellar system without being appreciably influenced by the attraction of the stars in their neighbourhood. He, moreover, found that this would not lead to any congestion of stars at the centre of the system, though the central density cannot be less than four times that near the sun.

These are the principal results of the work of Professor Eddington in the field of stellar motions. There is an alternative way of representing the phenomena of star-streaming, the ellipsoidal theory of Schwarzschild. But that does not affect the phenomena so thoroughly investigated by our Medallist; the two hypotheses, as he says, "express very nearly the same law of stellar velocities, but by the aid of different mathematical functions." A plausible suggestion has been made by Professor Turner, according to which the stars really move in very elongated orbits, something like the orbits of comets in our solar system, which might produce an effect like star-streaming. Whether we are prepared to accept this explanation or prefer to keep an open mind as to the interpretation of the observed motions, it is an undoubted fact that the stars have a strong tendency to move along a certain line in opposite directions rather than at right angles to that line. And this, though first announced by Kapteyn, was tested, discussed, and examined in all its bearings by Professor Eddington with remarkable thoroughness and skill, so that,

although there have been other investigators in this field, his name is more associated with this branch of sidereal astronomy than any other.

Up to the time when the spectroscope was first directed to the stars there was nothing known as to their physical constitution. The first attempt at classifying the stars according to their spectra was made by Secchi, whose four types were supposed to represent four stages in the life of every star, its temperature gradually falling from Type I., the hottest stars with spectra showing chiefly hydrogen lines, to Types III. and IV., stars of a reddish hue with strongly marked flutings. That a star might be growing hotter instead of always getting cooler does not seem to have struck anyone till J. Homer Lane in 1870 pointed out that when a gaseous body contracts from loss of heat its temperature will continue to rise, as the heat caused by the contraction is more than enough to keep the temperature from falling. Therefore the stars examined by the spectroscope could not all be getting cooler. But, as you remember, Norman Lockyer was the only astronomer who suggested that the stars of Types III. and IV. were not all verging on extinction, but that many of them are at the beginning of their career and in the act of getting hotter. It was not till 1913 (and on totally different grounds) that H. N. Russell and E. Hertzsprung showed that many well-known stars are in a diffuse and comparatively cool state, with rising temperature and a long life before them. Therefore the stars should be arranged in two series of rising and falling temperature respectively, so that a star will pass any temperature stage twice, ascending as a "Giant" star, of vast dimensions but of low density, and later as a shrunken but dense "Dwarf" of descending temperature. It was shown that the stars of Secchi's Types III. and IV., or, as we should now say, according to the Harvard classification, of M and N, exist in two sharply divided classes without any intermediate links.

These results of observation were strongly supported by the theoretical work of Professor Eddington in several papers, of which the earliest one goes over most of the ground and contains the most important results. It appeared in the *Monthly Notices* for 1916 November and 1917 June, and is entitled "On the Radiative Equilibrium of the Stars." This investigation on the interior of a star dealt chiefly with the giant stars, as the dwarf stars were supposed to be too dense to follow the laws of a perfect gas.

At any point inside the star two forces are acting, the weight of all the layers above pressing down on the gas below, and the elasticity of the layers below trying to expand and to force the upper layers up. Clearly there must be equilibrium, and these two tendencies just balance, since the star remains unchanged for hundreds of years; we know at any rate that more than a thousand of the stars visible to the naked eye are of the same magnitude now as they were 1800 years ago. This equilibrium is the principal clue by which we determine the amount of heat at different depths inside the star. Lord Kelvin once tried to find the age of the earth through the rate of the loss of heat from the surface. He knew the rate at which the temperature rises as we penetrate deeper into the earth, and he could measure the conductivity for heat near the surface. From these the corresponding rate of outflow of heat was

found. Professor Eddington tried in a similar way to determine the outflow of heat from a star, only it is the transparency instead of the conductivity of the stellar material that must be known, since in this case the heat passes out by radiation. But when determining this transparency, assuming that the mass of gas was simply acted on by its own gravity, an utterly absurd result was found.

It was therefore evident that there was another force affecting the equilibrium of a star, namely radiation-pressure. This light-pressure very possibly produces the tail of a comet, when it approaches the sun, by acting on the very minute dust-like particles forming the head of the comet and driving them off in the direction away from the sun. Inside a star it helps to bear the weight of the layers overhead, leaving less for the elasticity of the gas to bear. The proportion in which they share the weight can be calculated, and as a first approximation the same proportion holds good throughout nearly the whole interior, and depends only on the total mass of the star, not on the density or even on the chemical composition of the material. If we compute it for a number of globes of various dimensions, so that the nth globe contains 10^n grms., the following table gives some of the results:—

No. of Globe.	Light Pressure.	Material Pressure.
30	0·00000016	0·99999984
31	0·000016	0·999984
32	0·0016	0·9984
33	0·106	0·894
34	0·570	0·430
35	0·850	0·150
36	0·951	0·049
37	0·984	0·016
38	0·9951	0·0049

The stars are all of mass between the 33rd and 35th globes. The lightest known star would be just below the 33rd globe, the heaviest known is just above the 35th. The vast majority are between Nos. 33 and 34, just where the light-pressure becomes important. Professor Eddington thus gives a reason for the remarkable fact that the stars differ very little from one another as regards mass. With few exceptions they range from $\frac{1}{2}$ to 5 times the mass of the sun. Gravitation, if unresisted, would pile up material into globes of unlimited size. But this accumulation is limited by the disruptive force of radiation pressure, no doubt also by the centrifugal tendency of the star's rotation. This resistance only begins to be of any consequence when the mass has nearly reached that of the 33rd globe, and stars equal to or greater than that of the 35th globe are sure to break up sooner or later.

The gas of a typical giant star, with average density about equal to that of our atmosphere, consists of atoms rushing about in all directions with speeds up to 100 miles a second, always colliding and changing their courses. The energy of this motion constitutes a great store of heat, but the star contains also a store of another kind of heat, ethereal heat, or

ether waves like those which bring us the sun's heat across space. These waves are also in continual motion inside a star, colliding with the atoms and prevented by the material from escaping out into space except at a slow rate. Moreover, there are also inside a star great numbers of free electrons, unattached units of negative electricity, which have broken loose from the atoms at the high temperature here occurring. Hereby an idea may be formed as to the molecular weight of stellar material which is wanted for determining the constitution of a star. Supposing first that the temperature is so high that all the electrons have broken away, it then turns out that, no matter what element we take, the average weight of the ultimate particles is always about 2. For instance, an atom of sodium has separated into 12 particles, viz. 11 electrons and the nucleus, so that the atomic weight 23 is divided between 12 particles, the average weight of each being $23/12 = 1{\cdot}92$. The stars are, however, not hot enough to produce complete separation, but the view at which Professor Eddington has arrived in the last six months is, that for all except very heavy elements (heavier than iron) not more than two electrons will remain attached, and the molecular weight is probably between $2{\cdot}0$ and $2{\cdot}5$.

The next question is, how is the brightness of the star affected by the opacity of the material? The high temperature near the centre drives the heat outwards to the level of lower temperature, but the opacity of the material hinders this flow. The internal temperature distribution can be calculated, and we can observe the outward flow, which is the light and heat we receive from the star. In his presidential address last year on presenting the Gold Medal to Professor Michelson, Professor Eddington remarked that, thanks to Michelson, Capella now supplied the chief lacking constant in the radiative theory of stellar equilibrium, for which he had waited five years. He could now calculate the opacity of Capella, and it was found to be of the same order of magnitude as the opacity of terrestrial substances to X-rays measured in the laboratory. The terrestrial results indicate an extremely rapid change of absorption for slight changes of wave-length, while the astronomical results gave a nearly steady absorption coefficient. I must here pass over the theory proposed by Eddington to account for this discrepancy by investigating the conflict between detached electrons and atoms. It has met with opposition, though it was considered a strong point in its favour that the theory actually gave a value of the absorption coefficient agreeing with astronomical observation. On the other hand, on applying Kramer's theory of the continuous and line emission of radiation during the encounters of electrons with atoms to the case of Capella, Professor Eddington found (*M.N.* for January last) an absorption coefficient about one-eighth of that determined from astronomical observation. The problem of determining the absorption and emission in a star, a problem of extreme difficulty, is thus not completely solved yet.

There is a special class of giant stars, much more luminous than the average giants, the so-called Cepheid variables, which, though they show a periodic change of radial velocity, cannot be binary stars. It had been suggested by Plummer and Shapley that the light-changes are due to the pulsation of a single star, and that the varying radial velocity measures

the approach and recession of the surface in the course of the pulsation. To throw light on the phenomena of these peculiar variables, Professor Eddington investigated the theory of a star of which the equilibrium has been disturbed, so that it becomes a pulsating mass of gas, which remains spherical while expanding and contracting. He found that the theory can sufficiently well predict the periods both in the mean and individually from a knowledge of the type and absolute magnitude, and that it accounts for the advance of spectral type towards M with increasing luminosity. For stars of period greater than three days the central density is less than $\frac{1}{10}$ water, the condition approximating to that of a perfect gas throughout. The agreement of theory and observation is sufficiently good to allow us to hope that the true solution of the riddle of this class of variables has been found.

If this Address had been delivered in February last, I should at this point have turned to the third subject mentioned in the award. But, as you all know, at our meeting in March Professor Eddington read a very remarkable paper, which compels us to modify our conceptions of the Giant and Dwarf theory to some extent. Starting from the previously found value of the absorption coefficient, he calculated the difference of absolute magnitude between Capella and a number of stars, and thereby, using the observed data for Capella, the absolute magnitudes of these stars. More than half of the stars used are dwarf stars. It was found that their absolute magnitudes agreed remarkably well with the predicted magnitudes, which is not in accordance with the current idea that they are too dense to follow the law of a perfect gas, and that their faintness is due to deviation from the gas laws. But Professor Eddington points out that, owing to the high ionisation, stellar atoms have only about 1/100000 of the bulk of ordinary atoms, and therefore failure of the laws of a perfect gas is not to be expected till a density of 100000 times higher is reached. The conflict between the new results and the stellar evolution theory depends on whether we admit that the mass of a star gradually diminishes to any serious extent or not by radiation of energy. If a star is gradually burnt away, the only divergence between the new theory and the old one is that the diminishing brightness of the dwarf stars is due to decreasing mass and not to diminishing compressibility. Thus, although the internal temperature continues to rise, the conception of a rising and falling effective temperature is not abandoned.

Finally, we have to consider our Medallist's contributions to the theory of relativity. It appears that his attention was first drawn to the subject by de Sitter's three papers in the *Monthly Notices*, 1916, and in that and the following year he wrote a couple of short articles on Einstein's theory. At that time he was probably the only person in this country who had seen Einstein's principal paper. On 1918 February 1 he delivered a Friday lecture at the Royal Institution, and, being thereby tempted to go deeper into the subject, he wrote out a systematic treatment of the theory, with special reference to the astronomical tests, which was published as a Report to the Physical Society in the summer of 1918. This called public attention to the possibility of testing the theory by observing the deflection of rays of light from stars passing

close to the sun during the total eclipse of the sun on 1919 May 29. How two expeditions were sent out to observe this eclipse, one of them under Professor Eddington, and how the results confirmed Einstein's prediction, are facts which are too well known to need recapitulation here. I should only like in passing to remind you that it was the Astronomer Royal who first (in March 1917) pointed out the great opportunity of testing the theory, and that he was the prime mover of the two expeditions, who saw the thing through and overcame all the difficulties.

When he had finished the reduction of his plates of the eclipse, Professor Eddington wrote his book *Space Time and Gravitation*, which came out in 1920. It has been translated into French, Spanish, and German, which is no wonder, since it gives a singularly lucid exposition of Einstein's theory. And it does more than that. A mode of thought, a new field of exploration had been opened up, inviting anyone who had the necessary mental power to develop the results implied in Einstein's brief papers, and to find the consequences to which they led. This was done in the book, and thereby developments of the theory were made in various directions. In the last chapter, "On the Nature of Things," new ground is broken. As there stated, it looks like philosophy rather than ordinary science, owing to the necessity of using non-technical language. The ideas are developed as part of the regular physical theory in the book *The Mathematical Theory of Relativity*, which came out last year.

The point of this new departure is, that while in Einstein's theory it appeared that mass and momentum were *equal to* certain characteristics of the geometry of space time (components of curvature), our Medallist contends that mass and momentum are *the names under which we recognise* in physical experiments these geometrical characteristics. The first view introduces a law of nature which causes the equality; the second view reduces the so-called law of nature to a tautology. The mathematical theory of relativity is mainly an analysis performed on certain symbols; to apply it to the physical world we must make the connection by identifying certain "entities" appearing in the physics with certain entities appearing in the mathematical analysis. As the physical world and the mathematical theory run parallel to one another, the identifications occur at every point in the chain of the argument; but we must choose one set of identifications as the fundamental connection and deduce the others. Professor Eddington's view is, that the original identification of interval with the measured time and distance, although readily grasped, led to a topsy-turvy argument, and that better progress was made by identifying mass and momentum of matter with their corresponding analytical expressions and deducing all the other identifications from these.

Professor Eddington's principal contribution to the theory was published in 1921, entitled "A Generalisation of Weyl's Theory of the Electromagnetic and Gravitational Fields" (*Proc. Roy. Soc.*, A, vol. xcix.). On the basis of the principle of this paper, Einstein has since developed a theory, published in the Berlin Sitzungsberichte.

Thus Professor Eddington has worked with great success in three very

different fields of science. In that of sidereal astronomy he has elucidated the systematic motions of the stars. Passing into the borderland between astronomy and theoretical physics he has taught us what the stars are like inside. Finally, he has soared into the realm of relativity and helped considerably to develop the theory of Einstein.

Professor Eddington: I feel convinced that the award of our Gold Medal by the Council has never met with more unanimous approval from the Fellows than it does to-day. In handing you this token of our admiration and gratitude, let me add the earnest hope that you may long be enabled to enrich science as brilliantly as you have done in the past, and to be the "guide, philosopher, and friend" of astronomers for many years to come.

Spectroscopic and Magnitude Observations of o (Mira) Ceti, 1924.
By William J. S. Lockyer, M.A., Ph.D. (Plates 14, 15.)

It is well known that the long-period variable o Ceti (spectrum type M6*e*) does not reach the same degree of brightness at successive maxima. At these maxima the outstanding bright lines are those of hydrogen, H_δ generally outshining H_γ, and H_γ being considerably brighter than H_ζ or H_β. Other faint bright lines have been recorded as visible in the photographic spectrum at and after the epoch of maximum, and these increase in intensity and in number as the star becomes fainter while the hydrogen lines dim to near obscurity.

The importance of studying the spectrum of this star, when it comes within range, has been pointed out by many observers.

Thus Miss Agnes Clerke* (1903) stated: "The Potsdam plates were exposed during the low maximum of January and February 1896, when the variable scarcely exceeded the fourth magnitude; and this is a circumstance to be borne in mind, since there is reason to suspect that emission may differ, not only in degree but in kind, at light crises of different intensities."

Again, Baxandall† (1923) in an article on "The Bright Lines in the Spectrum of o Ceti" concluded with the statement: "Meanwhile, every opportunity should be taken to obtain further photographs of this interesting and complex stellar spectrum, with the object of augmenting, if possible, the present record of bright lines."

It was considered important, therefore, to take advantage of the approaching maximum of January last to secure as complete a series of photographs of the spectrum as possible, and it was hoped that this maximum might prove a bright one. The maximum magnitude attained was unfortunately only 4·9.

The results obtained may nevertheless be useful as a record of the appearance of the spectrum at this epoch, and so are brought together in the present paper.

* *Problems in Astrophysics*, p. 223. † *The Observatory*, **46**, 82.

The Light Curve at this Maximum.

A watch was kept on the star to note the time when its magnitude had sufficiently increased to obtain photographic records of its spectrum with the prismatic cameras of the Norman Lockyer Observatory. It was not until January 4 that the first photograph was taken with the 12-inch, and not until January 25 that the spectrum could be successfully photographed with the 9-inch.

According to Leon Campbell * the predicted date of maximum was 1924 January 29, but according to the December number of *L'Astronomie* (p. 532) the probable date of the maximum was given as February 26, with a probable magnitude of 3·2. The actual observed date of maximum, as recorded further on, was January 29, the date forecasted by Leon Campbell, and the observed maximum magnitude was 4·9.

The following table is the record of the observations for magnitude made at the Norman Lockyer Observatory between the hours of 7 p.m. and 10 p.m., the letters L. E. R. standing for Lockyer, Edwards, and Rimmer respectively. The fifth column gives the mean of the three observers.

While the Sidmouth observations were only considered as very approximate, application was made to Mr. Felix de Roy, the Director of the Variable Star Section of the British Astronomical Association, for more accurate values derived from the observations of members of his section. He very kindly forwarded a light curve extending from 1923 August to 1924 March 14, formed from 222 observations of the variable star section, but he considered this curve as "provisional," because, as he stated, "the observations do not include those of our observers in Australia, India, and South Africa, which, of course, carry a great weight and usually extend up to the end of March."

It is interesting to note that at the time of maximum phase much greater divergencies from the mean curve occur than at minimum or on the rise of the curve. Even his "first-class observations" differ by as much as half a magnitude (plus and minus) from his mean curve. It seems difficult to think that such observers can differ among themselves by as much as a whole magnitude in determining the magnitude of a star of about the fifth magnitude. It was noticed at Sidmouth that on nights during this phase the magnitude during an evening did not appear to be constant, but this was regarded as due to lack of experience in making the observations, rather than to actual changes of magnitude.

The question seems to arise: Does Mira at maximum undergo rapid oscillations of very short period of a few hours' duration? It would be interesting to make a detailed study of the observations on a large time scale to test this suggestion, but perhaps such an inquiry has already been made.

In the table Felix de Roy's mean values have been inserted against the dates of the Sidmouth observations, and the last column gives the number of photographs taken with the 12-inch prismatic camera on the

* *Harvard Coll. Obs. Circ.*, No. 250.

respective dates, a star (*) indicating that one of the photographs was secured with the 9-inch prismatic camera.

It will be seen from the table that the means of the Sidmouth observers did not differ very much from those of Felix de Roy, the greatest differences amounting only to two-tenths of a magnitude.

	Magnitudes.					Number of Spectra.
1924.	L.	E.	R.	Mean.	Felix de Roy.	
Jan. 4	6·8	6·4	6·4	6·5	6·41	1
6	6·3	...	...	6·3	6·15	...
9	6·0	5·9	6·1	6·0	5·82	1
10	5·7	5·7	...	5·7	5·70	1
14	5·4	...	5·5	5·5	5·35	...
19	5·2	5·3	...	5·3	5·10	...
25	4·9	5·1	5·2	5·1	4·93	2*
26	...	5·1	...	5·1	4·90	1
27	4·8	5·0	5·0	4·9	4·90	2*
28	...	4·9	...	4·9	4·90	...
31	4·6	5·0	5·0	4·9	4·90	...
Feb. 1	4·6	4·9	5·0	4·8	4·91	2*
2	...	...	5·0	5·0	4·91	...
6	4·5	...	5·0	4·8	4·93	...
14	4·7	...	5·0	4·9	5·02	1
16	...	...	...	...	5·05	1
23	...	...	...	...	5·13	1

Fig. 1 is a graphical representation of the above table. The continuous curve shows a portion of Felix de Roy's mean curve, while the crosses represent the magnitudes determined at Sidmouth. The vertical arrow represents the epoch of the observed (provisional) maximum as determined by Felix de Roy.

Magnitudes at Epochs of Maximum.

It was interesting to inquire how the magnitude at the present epoch compared with those that had previously occurred, because the spectrum about these times might be different at different epochs of maximum.

For this purpose reference was first made to a Harvard publication * in which are tabulated (with breaks) the epochs and magnitudes of the observed maxima up to 1905 February. In this publication the individual determinations of several observers are given for each epoch of maximum, but it was considered sufficient for the present purpose to adopt the mean in each case.

In 1916 Felix de Roy † published the magnitudes at the maximum

* *Harvard Coll. Ann.*, 55, 123.

† *Mem. d. Soc. d. Spett. Ital.*, 5, series 2*a*, p. 106, 1916.

phases from 1894 to 1915, and he has kindly supplied similar values when observed up to the recent maximum.

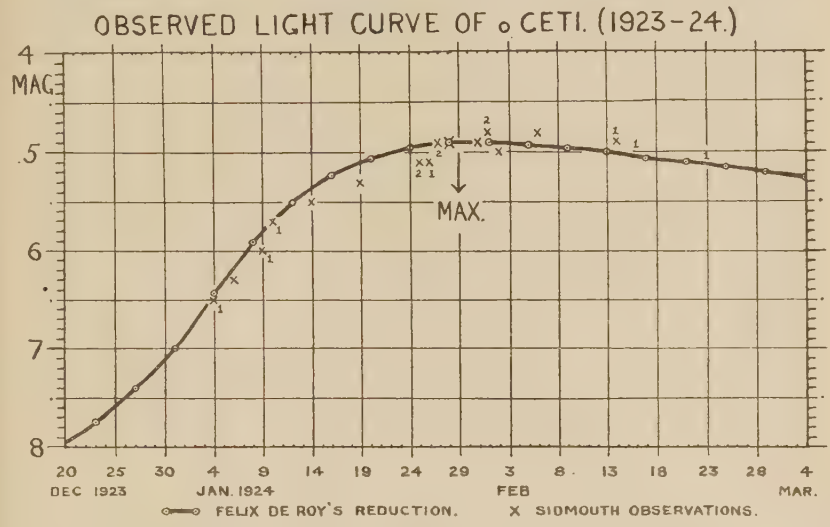


Fig. 1.

All the values to which reference above has been made have been plotted on the accompanying diagram (fig. 2).

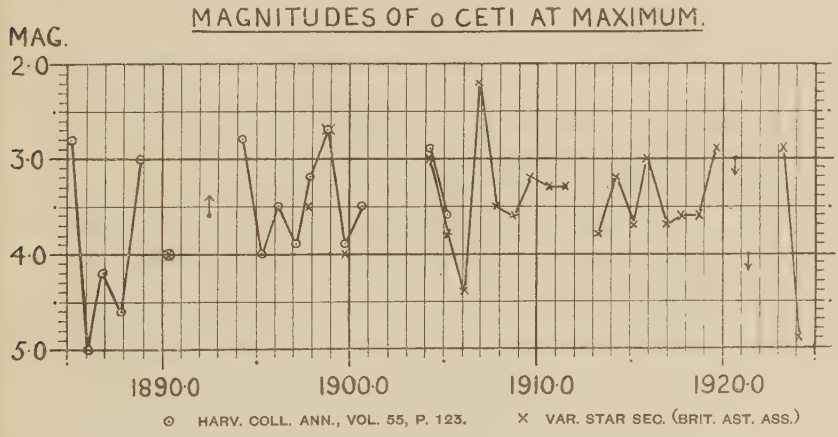


Fig. 2.

On this the Harvard values are denoted by circles and those by Felix de Roy by crosses. The small vertical arrows indicate that in the year 1902 Mira was brighter than magnitude 3·6, and in the years

1920 and 1921 fainter than magnitudes 3·0 and 4·0 respectively. The value for the year 1890 is stated * to have been about 4·0 magnitude.

This inquiry thus shows that the recent maximum was an extremely low one, in fact the lowest when any definite magnitude has been assigned since 1886.

The Prismatic Cameras Used.

Arrangements were made to photograph two series of spectra, one with the 12-inch and another with the 9-inch prismatic camera. The former instrument (prism 20° angle) has a dispersion of 21·4 mm. and the latter (prism 45° angle) of 46·0 mm. between H_β and H_δ. Messrs. D. L. Edwards and W. B. Rimmer secured the photographs with the 12-inch instrument, while the writer took those with the 9-inch instrument. Thirteen photographs altogether were obtained, covering the period January 4 to February 23. When Mira was about its maximum brightness the exposures with the 12-inch and 9-inch were 95 and 118 minutes respectively on January 27, and 120 and 105 minutes respectively on February 1. Similar plates were used in both instruments, namely, Ilford Iso Zenith (H. & D. 700).

The Intensities of the Hydrogen Lines.

The peculiarity of the behaviour of the bright hydrogen lines in this star is well known. Instead of the lines decreasing in brightness as the violet is approached, quite a different law for their intensities is in operation.

In all the photographs taken near or at the time of the recent maximum, the order of the intensities of the lines recorded, commencing with the strongest line, was as follows:—

In the case of the 12-inch prismatic camera H_δ, H_γ, H_ζ.
,, ,, 9-inch ,, ,, H_δ, H_γ, H_ζ, H_β, H_η.

It will be noticed that H_ϵ is missing, and this fact accords with observations made by other observers at most previous maxima. Its absence is generally credited to the masking of the line by the strong and broad calcium absorption at λ3968·6.

The fact that the recent maximum was of low intensity (magnitude 4·9) may account for the general comparative feebleness of the hydrogen radiation in relation to the dark flutings.

That the above sequence of intensities is maintained at all epochs of maximum brightness is, however, not a certainty, even allowing for the facts that instruments for recording the spectrum are different and that various kinds of photographic plates have been used. It is therefore not without interest to record the order of the intensities of the hydrogen lines in spectra taken at Sidmouth with the same prismatic camera (but different photographic plates) at this and previous maximum epochs together with the observations of other observers when the information is available.

In the following table the observations include ten maxima, covering

* *The Observatory*, **30**, 144, 1907.

Max. of Mira.			Month of Obs.	Hydrogen Lines in Order of Intensity.	Authority.	Reference.
Mag.	Year.	Month.				
2·2	1906	Dec.	Dec.	$H\delta$ $H\gamma$ $H\beta$	Slipher	*Astrophys. J.*, 25, 236, 1907.
2·2	,,	,,	,,	$H\delta$ $H\gamma$ $H\zeta$ $H\beta$ $H\eta$ $H\iota$	Cannon	*Ann. Harv. Coll. Obs.*, 56, 104, 1912.
2·2	,,	,,	,,	$H\delta$ $H\gamma$ $H\beta$	Plaskett	*Jour. R.A.S. Can.*, 1, 45.
2·7	1898	Oct.	Oct.	$H\gamma$ $H\delta$	Sidgreaves	*M.N., R.A.S.*, 59, 509, 1899.
2·9	1919	Aug.	Aug.	$H\delta = H\zeta$ $H\zeta$ $H\gamma$ H $H\eta$ $H\theta$	Shane	*Lick Obs. Bull.*, No. 335, p. 134, 1921.
2·9	1923	Mar.	Mar.	$H\delta - H\gamma$ $H\gamma$ $H\beta$ $H\zeta$	Lockyer	From Sidmouth Photographs.
3·0	1915	Dec.	Dec.	$H\delta$ $H\gamma$ $H\beta$ $H\zeta$ $H\eta$ $H\theta$	,,	,, ,, ,,
3·2	1897	Nov.	Nov.	$H\delta = H\gamma$ $H\gamma$ $H\beta$?	Sidgreaves	*M.N., R.A.S.*, 58, 345, 1898.
3·6	1905	Feb.	Feb.	$H\delta$ $H\gamma$ $H\zeta$ $H\iota$ $H\eta$	Cannon	*Ann. Harv. Coll. Obs.*, 56, 102, 1912.
3·7	1915	,,	,,	$H\delta$ $H\gamma$ $H\zeta$ $H\eta$ $H\beta$	Lockyer	From Sidmouth Photographs.
3·7	,,	,,	Jan.	$H\delta$ $H\gamma$ $H\zeta$ $H\eta$ $H\iota$ $H\theta$	Merrill	*Pub. Ast. Obs. Mich.*, 2, 51, 1916.
3·8	1902	Jun.	Jun.	$H\delta$ $H\gamma$ $H\zeta$ $H\beta$ $H\eta$	Stebbins	*Lick Obs. Bull.*, No. 41, p. 94, 1903.
4·9	1924	Jan.	Jan.	$H\delta$ $H\gamma$ $H\zeta$ $H\beta$ $H\eta$	Lockyer	From Sidmouth Photographs.
				$H\delta$ $H\gamma$ $H\zeta$ / $H\beta$ $H\beta$ / $H\eta$ $H\eta$ / $H\iota$ $H\iota$ / $H\theta$ $H\iota$ /		

the years from 1897 to 1924, and they are arranged in order of brilliancy of the maxima, commencing with the brightest to show whether the sequence of intensities of the hydrogen lines varied at all with diminution of maximum brightness of Mira. The table is self-explanatory except that the brightest of the hydrogen lines commences on the left. References to the authorities from which the facts have been taken are given in the last column.

As no systematic difference seemed at first sight to be apparent, the table was divided into three groups, namely, one group combining the magnitudes 2·2 to 2·9; the second group including the magnitudes 3·0 to 3·8; and the last group containing only the magnitude 4·9. All the groups showed practically the same general sequence, namely,

$$H_\delta,\ H_\gamma,\ H_\zeta,\ H_\beta,\ H_\eta,\ H_\iota,\ H_\theta.$$

At the bottom of the table is given a summation of the general order of the lines in intensity, the second and third rows showing the second and third frequencies of appearance in the respective columns above them. When a line has appeared in a previous column it has been made to take precedence in the order of intensity of one which has not been recorded there.

It will be seen that occasionally H_ζ changed places with H_β and H_η with H_ι, but there seemed to be no systematic alteration with magnitude. The changes of the first pair H_ζ and H_β could easily be accounted for by the use of plates of different sensitiveness, as those lines lie towards the extremes of the spectrum where the sensibility begins to decrease rapidly. H_β, however, lies more in the photographic region of the spectrum than H_ζ, so that on the whole H_β should normally be more intense than H_ζ; but it is not.

Attention may be drawn to the fact that a short time (August 30) before the bright maximum of 1898 October, when Mira reached the magnitude of 2·7, Campbell * found that the bright lines H_δ and H_γ were broken up into three unequal components. Wright † secured a number of photographs of the H_γ line at the maximum of 1899 (magnitude 3·2) showing this line to be double not triple. A critical examination of the negatives taken with the 9-inch prismatic camera at the recent maximum, which has considerable dispersion, but not approximating to that employed by Campbell, displays simply monochromatic images of those lines. It is quite possible that the production of multiple-hydrogen lines may be restricted to brilliant maxima only.

Plate 14 reproduces two of the Sidmouth spectra of Mira taken on 1924 February 1 and 1914 December 21, together with two comparison spectra, namely, α Canis Minoris and α Herculis.

The Absorption Lines.

The absorption lines, as is well known, correspond to those in Secchi's third-type stars, and in the Henry Draper Catalogue the star is classed

* *Astroph. J.*, 9, 32.
† *Lick Obs. Bull.*, No. 183, p. 60.

1. α Can. Min. 2. ο Ceti, 1914 Dec. 21. 3. ο Ceti, 1924 Feb. 1. 4. α Her.

as Md. The lines are necessarily most conspicuous in the region of the spectrum, where they are not intermingled with the dark titanium oxide flutings, this region lying approximately between H_γ and H_η, allowing for the falling off in intensity beyond the latter line.

A very full list of the absorption lines has been published by Stebbins * from the negatives he secured in 1902, so no attempt was made to deduce the wave-lengths of all the lines in the Sidmouth photographs.

The procedure here adopted was simply to select the most prominent lines and determine their wave-lengths and origins and compare them with the strongest absorption lines previously recorded at the maximum phase.

Stebbins gives a brief summary of the number of dark lines of the different elements observed in his spectra, and this list, given below, is compared with elements definitely found to be present in the Sidmouth photographs.

Sidmouth.	Stebbins.	Sidmouth.	Stebbins.
Ca	Ca 6	Sr	Sr 1
Mn	Mn 3 (?)	Cr	Cr 9
Fe	Fe 11	Ti	Ti 2 (?)
Al	Al 2	...	V 11

According to Stebbins the presence of manganese is doubtful, but in the Sidmouth photographs the quartet of manganese was found to be a very prominent feature of the spectrum (see Plate 15). In the case of titanium, the presence of which he also questions, four of the conspicuous dark lines selected for wave-length determination were found to be titanium lines, three of which are very strong arc lines, namely, λλ's 4024·8 (Int. 8), 4544·8 (Int. 8), and 4667·4 (Int. 9).

Plaskett,† at the very bright maximum of 1906 December (magnitude 2·2), recorded that at least one-fourth of the identifications of the prominent absorption lines measured in two spectra appeared to be due to titanium.

While Stebbins gives eleven lines for vanadium, none of the selected lines in the Sidmouth photographs were identified as due to this element. A special examination of the photographs for these lines showed that many of the strongest were present, but not sufficiently prominent to be among the few lines selected for their intensity and wave-length determination.

In Plate 15 the chief absorption lines mentioned above are indicated in diagrammatic form with the star α Canis Minoris (Type F5) for comparison. Two photographs of Mira taken this year are shown, one with the 9-inch prismatic camera on January 27 and the other with the 12-inch prismatic camera on February 1. The original photographs were on very different scales, but they have been enlarged to the same scale and broadened by the clepsydra apparatus.

* *Lick Obs. Bull.*, No. 41, p. 81, 1903
† *Jour. R.A.S. Canada*, 1, 58, 1907.

Lines between H and K of Calcium.

All the spectra show the calcium lines at H and K as very broad and λ4227 as a prominent absorption line. The best photographs taken with the 12-inch and 9-inch prismatic cameras indicated what appeared at a first glance to be strong reversals of H and K, or bright lines in the broad H and K absorption bands.

A more careful examination showed that these bright lines were by no means central on these bands. The bright line near H was well to the more refrangible side of the centre of H, and the bright line at K well towards the less refrangible side of the centre of K.

An examination of negatives taken about the times of previous maxima of Mira at the epochs 1915 February and December showed similar apparent reversals at H and K (see Plate 14, No. 2).

It was ultimately concluded, however, that these were not bright lines at all, but small strips of continuous spectrum sandwiched between absorption lines. Measures were made to determine the wave-lengths of the absorption lines and they were found to be λλ's 3944·4 and 3962·5. These values corresponded sufficiently close to those of λλ's 3944·1 and 3961·7 to indicate that they were the strong aluminium lines conspicuous in the chromosphere and fainter in γ Cygni.

Both these lines are recorded as absorption lines by Stebbins,* who describes their intensities and character as five and wide respectively.

Plate 14 reproduces the spectra of o Ceti for the dates 1914 December 21 and 1924 February 1, showing the apparent reversals at H and K. The comparison with α Herculis given at the bottom illustrates their identification with the two aluminium lines.

The Absorption Flutings.

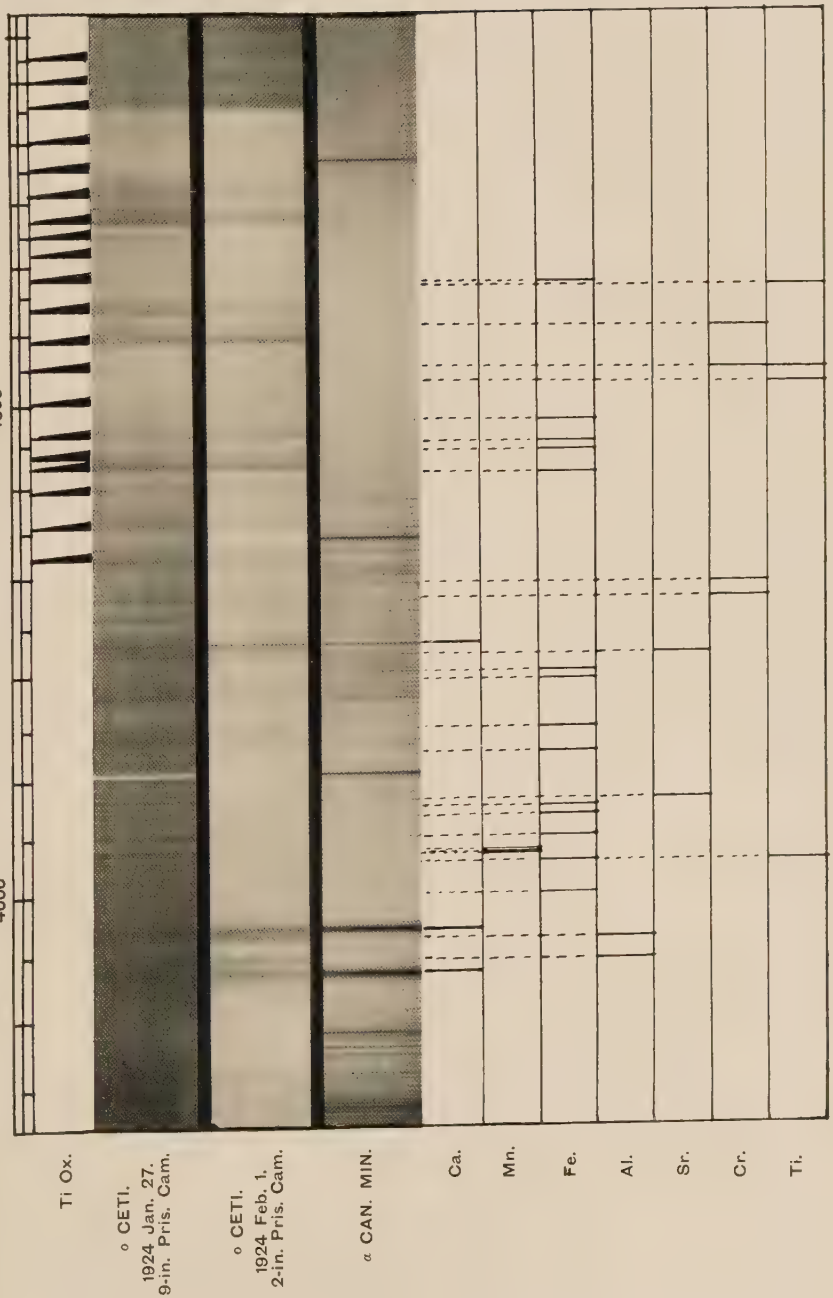
Apart from the bright hydrogen lines, the pronounced dark flutings are the most conspicuous objects in the spectrum. Their origin, discovered by Fowler † in 1904, is titanium oxide, and he gives in his paper the wave-lengths of the several flutings from λ5847·2 to λ4352·9.

The present photographs of Mira exhibit all these flutings from λ4352·9 to λ5167·5, after which the sensitiveness of the plates used falls off rapidly. All these flutings are indicated at the top of Plate 15 in relation to the star's spectrum.

Other Bright Lines.

A search was made for bright lines other than those of hydrogen, and in the first instance the best negative (No. 425, 1924 January 27) taken with the 9-inch prismatic camera was compared with a good negative of α Cassiopeiæ (Type Ko) for wave-length determination. A close examination showed that no bright lines could be definitely distinguished. It was true there were markings which at first sight appeared as bright lines, but comparison with the negative of α Cassiopeiæ

* *Lick Obs. Bull.*, No. 41, p. 85. † *Roy. Soc. Proc.*, 73, 219.


4000
4500
5000
Ti Ox.
o CETI.
1924 Jan. 27.
9-in. Pris. Cam.
o CETI.
1924 Feb. 1.
2-in. Pris. Cam.
α CAN. MIN.
Ca.
Mn.
Fe.
Al.
Sr.
Cr.
Ti.

indicated that they were more probably the interspaces between the absorption lines which were seen to be present.

Similar remarks apply to the examination of the best 12-inch prismatic camera negative (No. 3115, 1924 February 1) using α Canis Minoris as a comparison spectrum.

An examination was also made of a negative (No. 615, 1915 December 11) taken with the 12-inch prismatic camera at about the time of the bright maximum (magnitude 3·0) in 1915. In this the hydrogen lines are more pronounced relatively to the flutings and H_β shows up strongly. When this photograph was compared with that of α Canis Minoris it was found most difficult to differentiate between apparent bright lines and interspaces between absorption lines.

A conclusive argument against the appearance of any bright lines was shown by comparing the spectrum of Mira with that of α Herculis (Type Md). Line for line and interspace for interspace were identical throughout both spectra, and the intensities of both were remarkably similar (see Plate 14).

As it is well known that α Herculis has no bright lines in its spectrum, the same must be true for that of Mira in this particular instance.

At the very bright maximum of 1906 December (magnitude 2·2) Plaskett* records the presence of eight bright lines, but he also refers to the great difficulty of being quite certain of their nature, for he says "it seems hardly possible that they can be narrow strips of continuous spectrum left unabsorbed . . ." and "it may be said on the contrary, however, that they have not been identified with any one element, and that the nearest identifications are of elements which have the most pronounced lines in the absorption spectrum." He regards three of these lines as certainly emissive, but of the others he does not feel so sure.

Stebbins,† from photographs taken three weeks after the predicted time of maximum in 1902 June, found some very faint bright lines, which afterwards increased in number and intensity as the minimum was approached, while the hydrogen lines almost vanished. This fact seems to suggest that at a time of maximum the bright lines in the spectrum of Mira are only those due to hydrogen.

Summary.

1. The paper describes the spectroscopic and magnitude observations of o Ceti made during the last epoch of this star's maximum (1924 January 29).

2. The Sidmouth magnitude determinations are compared and agree well with those deduced by Mr. Felix de Roy from 222 observations made by the variable star observers of the British Astronomical Association.

3. The maximum of o Ceti reached a very low magnitude, namely, 4·9, the lowest since 1886 so far as is recorded.

* *Journ. R.A.S. Canada*, 1, 55, 1907.
† *Lick Obs. Bull.*, No. 41, p. 91, 1903.

4. The bright hydrogen lines decreased in intensity in the following order, H_δ, H_γ, H_ζ, H_β, H_η, H_ι, H_θ.

5. This order seems to remain constant however bright *o* Ceti may be at maximum.

6. The apparent reversals of the calcium lines at H and K are due to strips of continuous spectrum lying between the two strong absorption lines of aluminium and the H and K lines.

7. The chief absorption lines are due to the elements usually recorded, namely, Ca, Mn, Fe, Al, Sr, Cr, Ti, V. Manganese and titanium, about which some uncertainty has been expressed, are certainly present.

8. All the flutings of titanium oxide tabulated by Fowler, which fall within the range of the photographic plate used, are recorded in the spectra.

9. No bright lines other than hydrogen could be recorded with certainty.

Description of the Plates.

Plate 14. Two photographs of *o* Ceti taken with the 12-inch prismatic camera in 1914 December 21 and 1924 February 1 at epochs of maximum are shown. The spectra of α Herculis and α Canis Minoris are given for comparison purposes.

Plate 15. Two photographs of *o* Ceti are illustrated. The upper one taken with the 9-inch prismatic camera in 1924 January 27, and the lower one with the 12-inch prismatic camera in 1924 February 1. The comparison spectrum is that of α Canis Minoris.

The plate shows the identification of the chief absorption lines in *o* Ceti with the elements stated, while the titanium oxide flutings are also indicated.

Absorption Bands in Celestial Spectra. By F. E. Baxandall, A.R.C.Sc.

(*Communicated by the Director of the Solar Physics Observatory, Cambridge.*)

It is well known that the G type of stellar spectra is the earliest type to show bands or flutings as distinguished from lines. The cyanogen band with chief head at $\lambda 3883{\cdot}6$ was the first to be recognised in the solar spectrum. Stellar spectra of later types than the sun (G type) all show banded spectra of some form or other, and it is generally recognised now that these cannot be placed in a continuous sequence following on after O, B, A, F, G groups, but that after G there seem to be at least two branches which take the orders G—R—N to the spectra showing strong carbon absorption, and G—K—M to spectra showing strong TiO_2 absorption. Recently another class S has been added, an account* of which type has been given by Merrill. These, which show marked bands due to neither carbon nor TiO_2, are referred to by Merrill as being probably more closely related to M than to N stars.

Sunspot spectra have also been found to have marked banded absorption, the three most prominent elements represented here being

* *Ast. Phys. Journ.*, 56, 457.

titanium (oxide), magnesium (hydride), and calcium (hydride). There are many other local regions of absorption in sunspot spectra which strongly suggest heads of bands. As in some of the later stellar types there are prominent bands which have not yet been identified, it has been thought desirable to bring together in order of wave-length all the apparent bands whose positions in celestial spectra have been recorded, or are recognisable from published reproductions of the spectra.

It seems rather a curious fact that although many chemical elements give banded spectra as well as line spectra, there should be such a disparity between the number of elements represented in celestial spectra by their lines, and the number whose banded spectra have been traced. So far as is known, the only elements represented in celestial spectra by their bands are carbon (Swan spectrum, hydrocarbon, cyanogen) in various later-type stars: titanium (oxide) in K and M type stars and in sunspots: magnesium and calcium (hydrides) in sunspots: and zirconium (oxide) in S type stars. These identifications involve compounds of only eight of the chemical elements. A much lengthier list of simple elements is known to be represented in celestial spectra.

The bands of zirconium oxide have recently been identified with bands in S type spectra, the new type whose features have been recorded by Merrill. Shortly after Merrill's paper appeared the present writer was struck by the similarity in wave-length of some of the S type bands, and the published wave-lengths of zirconium bands. It was suggested to Mr. Merrill that the stellar spectra be more closely investigated with reference to the possible connection of the two sets of bands. In a more recent note * Merrill records that he has directly compared the spectra of the S type stars with the ZrO spectrum, with the result that all the bands recorded for the latter—thirteen in number —in the region λ6229 to λ6612 are found in the stellar spectra. Further details he has reserved for a later paper.

The chief records utilised in compiling the list of bands given in the following table are: the Fraunhofer spectrum as shown in Rowland's maps: Hale's sunspot maps: Stebbins' spectra of *o* Ceti and other M type stars: Rufus's spectra of R type stars: Hale, Ellerman, and Parkhurst's fourth-type stars: Merrill's S type stars: and Wright's spectrum of R Cygni (an S type star). Cometary spectra have not been included, as the bands in this type are all due to carbon in some form or other, and, with the exception of the low-pressure carbon oxide bands which Fowler found in the spectra of comets' tails, are all found in one or other of the types of celestial spectra included in the table in this paper. The ammonia bands traced by Fowler in the solar spectrum have also been omitted from the table.

The bands inserted in the table as being apparently present in sunspot spectra have been taken from Hale's maps. The writer has used his own discretion as to what he judges to be heads of bands, but it is possible that some may be included which are in reality close groups of lines appearing in the spot spectrum and not in the Fraunhofer spectrum. Probably no two investigators going through Hale's maps would pick out exactly the same set of apparent bands.

* *Pub. Ast. Soc. Pacific*, 35, 217, 1923.

The bands hitherto unidentified have been compared as to position with the bands recorded for various chemical compounds. In cases where it is thought there may be some connection between the stellar and terrestrial bands the possible origin has been inserted in the last column. It can be said at once, however, that there is little evidence for the identification of the great majority of bands included in the table other than those previously ascribed to carbon, titanium, magnesium, calcium, and zirconium.

The table is self-explanatory. For a few of the bands it has been found necessary to add notes on points which could not be adequately explained in the last column.

The letters in brackets (*a*, *b*, etc.) at the head of the various columns apply to the references given at the end of the paper. The letters R and V in the second column indicate the direction (towards red or violet) of the gradual fading of the bands. The crosses (×) indicate the presence of the particular bands in the various stellar types.

Apparent Absorption Bands in Celestial Spectra.

λ.	Direction of Fading: R, Red; V, Violet.	(*a*) Sun-spot Spectra, Hale's Maps.	(*b*) Solar Spectrum and G Type Stars.	(*c*) M Type Stars, o Ceti, etc. (Stebbins).	(*d*) R Type Stars (Rufus).	(*e*) N Type Stars (Hale, Ellerman, and Parkhurst).	(*f*) (*g*) S Type Stars (Merrill) and R Cygni (Wright).	Remarks as to Probable Origin, etc.
3850·9	V		×					CN.
3855·1	V		×					CN.
3861·9	V		×					CN. Band 4. (Note 1.)
3871·5	V		×					CN.
3883·6	V		×					CN.
4216·1	V		×		×			Very weak in sun, strong in R stars. C.N. 1st head of band 3.
4313·4	R			×				(Note 2.)
4314·26	V		×					Hydrocarbon band. Main head 4314·26 well developed in Fraunhofer spectrum, but generally stronger in R stars. (Note 3.)
4314·7	V				×			
4352·7	R			×				TiO_2.
4380·6	V					×		Possibly Swan band, main head 4381·93.
4395·0	R			×				TiO_2.
4395·6	V				×			
4409·4	V				×			
4421·5	R			×				TiO_2.
4462·1	R			×				TiO_2.
4503·2	V					×		CN., 6th head, band 2.

Apparent Absorption Bands in Celestial Spectra—continued.

λ.	Direction of Fading: R, Red; V, Violet.	(*a*) Sun-spot Spectra, Hale's Maps.	(*b*) Solar Spectrum and G Type Stars.	(*c*) M Type Stars, o Ceti, etc. (Stebbins).	(*d*) R Type Stars (Rufus).	(*e*) N Type Stars (Hale, Ellerman, and Parkhurst).	(*f*) (*g*) S Type Stars (Merrill) and R Cygni (Wright).	Remarks as to Probable Origin, etc.
4504·8	R			×				TiO_2.
4515·0	V					×		CN., 5th head, band 2.
4532·6	V					×		CN., 4th head, band 2.
4547·9	R			×				TiO_2.
4553·4	V				×			
4555·3	V					×		CN., 3rd head, band 2.
4578·4	V					×		CN., 2nd head, band 2.
4584·1	R			×				TiO_2.
4608·8	V					×		CN., chief head of band 2 (4609–4503).
4625·8	R			×				TiO_2.
4640	R						×	Given by both Merrill and Wright. ? Zr band. (Note 4.)
4668·6	R			×				TiO_2.
4697·2	V					×		3rd head, 4th Swan band.
4713·2	R			×				
4716·5	V					×		2nd head, 4th Swan band.
4737·1	?R							α Herculis (Stebbins). (Note 5.)
4737·6	V				×			1st head, 4th Swan band.
4738·6	V					×		
4745·1	V				×			Probably carbon. (Note 6.)
4745·2	V					×		
4752·8	V				×			(Note 7.)
4753·1	V					×		
4759·0	R			×				TiO_2.
4803·2	R			×				TiO_2.
4841·8	R			×				
4848·4	R							ρ Persei (Stebbins). TiO_2. (Note 8.)
4855·5	V					×		In 132 Schjellerup.
4871·6	R					×		In 152 ,,
4890·3	V					×		n 78 ,,
4892·1	V				×			
4901·1	R					×		In 78 ,,
4910	R					×		In 152 ,,
4953·1	R			×				TiO_2.

Apparent Absorption Bands in Celestial Spectra—continued.

λ.	Direction of Fading: R, Red; V, Violet.	(*a*) Sun-spot Spectra, Hale's Maps.	(*b*) Solar Spectrum and G Type Stars.	(*c*) M Type Stars, o Ceti, etc. (Stebbins).	(*d*) R Type Stars (Rufus).	(*e*) N Type Stars (Hale, Ellerman, and Parkhurst).	(*f*) (*g*) S Type Stars (Merrill) and R Cygni (Wright).	Remarks as to Probable Origin, etc.
5052·8	R	×						
5058·8	R	×						
5077·2	V	×						
5085·6	V	×						
5089·4	V	×						
5093·0	V	×						
5096·4	V	×						
5101·9	V	×						
5105·2	V	×						
5112·0	V	×						
5113·5	V	×						
5126·2	V	×						
5150·2	V	×						
5155·1	V	×						
5165·9	R			×				TiO_2.
5167·9	V					×		1st head, 3rd Swan band. Laboratory λ5165·3.
5211·0	V	×						Head of magnesium hydride band.
5307·2	R			×				TiO_2.
5357·7	R			×				TiO_2.
5409·0	R	×						
5438·3	R			×				
5446·8	R			×				TiO_2.
5457·0	R	×						
5459·5	V							
5496·9	R			×				TiO_2.
5505·4						×		4th head, 2nd Swan band.
5510·7	R	×						
5520·5	V	×						
5527·5	R	×						
5543·5	V					×		3rd head, 2nd Swan band. Laboratory λ5540·9.
5550	R						×	Shown in Wright's reproduction of R Cygni. ? ZrO band. (Note 4.)
5566·8	V	×						
5587·7	V					×		2nd head, 2nd Swan band. Laboratory λ5585·5.

Apparent Absorption Bands in Celestial Spectra—continued.

λ.	Direction of Fading: R, Red; V, Violet.	(*a*) Sun-spot Spectra, Hale's Maps.	(*b*) Solar Spectrum and G Type Stars.	(*c*) M Type Stars, o Ceti, etc. (Stebbins).	(*d*) R Type Stars (Rufus).	(*e*) N Type Stars (Hale, Ellerman, and Parkhurst).	(*f*) (*g*) S Type Stars (Merrill) and R Cygni (Wright).	Remarks as to Probable Origin, etc.
5626·0	V	×						
5632·5	V	×						
5638·8	V					×		1st head, 2nd Swan band. Laboratory λ5635·4.
5659·8	V	×						
5721·4	R					×		In 115 Schjellerup.
5758·2	R	×						
5772·5	V	×						
5853·4	V	×						
5859·3	V	×						
5883·8	V	×						
5906·4	V	×						
5906·7	R	×						
5936·0	V	×						
6221·8	V	×						
6240·8	V	×						
6275·5	R	×						TiO_2.
6342	R						×	From Wright's spectrum light curve, R Cygni. ? Zr band. (Note 4.)
6382·2	V	×						Calcium hydride.
6389·4	V	×						,, ,,
6392·0	R	×						
6406·6	V	×						
6470	R						×	Given by both Merrill and Wright. Zr band.
6651·2	R	×						TiO_2.
6680·8	R	×						TiO_2.
6724·8	V	×						
6781·2	R	×						TiO_2.
6814·8	R	×						TiO_2.
7054·5	R	×						TiO_2.
7087·8	R	×						TiO_2.
7092·9	R	×						TiO_2.
7101·5	V	×						
7102·7	R	×						
7125·6	R	×						TiO_2.

Notes on some of the Absorption Bands.

(Note 1.) These form the fourth band of the cyanogen spectrum. They are conspicuous in the solar spectrum and in G type stars. Their presence in later types than G is doubtful, the continuous absorption in the region of the band being such as to make it impossible to decide the point. There is little or no evidence of their presence in types earlier than the G type.

(Note 2.) This *o* Ceti band is so near the position of the chief head of the hydrocarbon band that the first impulse is to associate the measured edge with the edge of the HC. band (4314·26), which forms such a conspicuous feature of the Fraunhofer spectrum and of the class R stars (with one exception, B.D. $-10^{\circ}\cdot5057$) discussed in much detail by Rufus. But as Stebbins assumes the absorption bands, whose heads he has measured, shade to the red (as the great majority—TiO_2 bands—undoubtedly do), to ascribe the 4313·4 to hydrocarbon would rather imply that the latter is an emission band in *o* Ceti. There is, however, no satisfactory evidence of other emission bands occurring in *o* Ceti, accordingly the band is entered in the table as R and it must at present be classed as unknown. There appears to be no record of a TiO_2 band at this wave-length.

(Note 3.) This band, as it occurs in the solar spectrum, was shown by the Cambridge observers * to be identical with the well-known hydrocarbon band whose main head is 4314·26. A comparison of Rowland's solar lines of unknown origin in the region 4314 to 4180 with the individual structure lines of the hydrocarbon band as published by Eder and Valenta clearly shows the identity of the two sets all along the spectrum. This band is also well developed in the G type stars but less so in the K type. In most of the R stars it forms a conspicuous feature. In the N stars there is so much continuous absorption beyond λ4400 on the violet side that it is impossible to say whether the hydrocarbon band exists in that type.

There is a subsidiary hydrocarbon band starting at λ4389·96 and a detached group of lines λλ4323·0–4324·5 which both occur in the solar spectrum, and probably also in R type stars, but in the latter case there is difficulty in locating the head of the band owing to the proximity of other lines.

(Note 4, band 4640.) This is given both by Merrill in his S type stars and by Wright in his spectrum of R Cygni. In Merrill's reproductions it is quite a conspicuous band. There is a zirconium oxide band at this wave-length, and as in other regions of the S type spectra zirconium bands have been shown to be present, it seems probable that the 4640 band may also be due to that element.

(Band 5550.) In Wright's reproduction of the R Cygni spectrum there is a strongly marked band with violet edge at λ5550 as nearly as can be estimated from his wave-length scale. As there is also a zirconium band recorded at the same wave-length, it seems probable that the stellar and laboratory bands are identical.

(Band 6342.) A band at this wave-length was indicated by Wright's light-curve of the spectrum of R Cygni. There is a zirconium band near the same position, and as Merrill has recorded † the presence in S type spectra of all the zirconium bands in the region λ6229 to λ6612, Wright's band is possibly one of these.

(Note 5.) With regard to this band λ4737·1 in α Herculis, Stebbins says: "The head at λ4737·1 measured in α Herculis may correspond to the fourth carbon band λ4737·2. These two coincidences [he has previously mentioned that of his *o* Ceti head λ5165·9 with the third carbon band λ5165·3] are the only ones I have found, and we certainly need more evidence before drawing any conclusions from them." It was, however, conclusively shown later by Fowler that all the main bands in *o* Ceti and kindred stars such as α Herculis could be accounted for by TiO_2 bands without carbon bands coming into the question at all.

* *M.N., R.A.S.*, **76**, 640.
† *Pub. Ast. Soc. Pac.*, **35**, 217, 1923.

Stebbins' measures are of heads of apparent "absorption" bands shading to red, so that if the measured head 4737·1 be assumed to be that of the carbon band 4737 shading to violet, it would necessarily imply that this band is an emission band. There is, however, no satisfactory evidence of the carbon bands occurring in this type of spectrum either as emission or absorption, and Stebbins' α Herculis band must be attributed to some source other than carbon.

(Note 6.) In some of the R and N type spectra, in addition to the head of the Swan band at λ4737 there appears an extraneous line to it near λ4745. This sometimes takes the form of a strong absorption line clearly separated from the head of the Swan band. In certain N type stars, however, notably B.D. +46°·1817,* this line appears to be the red-end head of a strong fluting which blocks out completely the head of the 4737 band. So far as the writer knows nothing has been recorded in the various publications on carbon spectra which would account for this extra head. A search through the Kensington and Cambridge series of carbon spectra has shown that several photographs show such a line, and its wave-length is 4745. There appears to be little doubt that this is in a position identical with that of the extra head noted in the R and N type spectra.

(Note 7.) In exactly the same way as the line or band mentioned in Note 6 sometimes forms a second head outside the 4737 Swan band, there is in some of the R and N types a similar extraneous line near λ4752 which is apparently associated with the other two heads. In the N star B.D. +46°·1817 referred to in Note 6 the line 4752 appears to be a strong absorption line. In the R type star B.D. +57°·702, however, 4752 is the head of a very dense absorption band which, fading off to the violet, shows such strong continuous absorption that the two heads at 4745, 4737 seen in many other stars of these types are completely blocked out.

Search through the published records of carbon lines and flutings and through the many carbon photographs in the Kensington and Cambridge series has failed to produce any evidence on which the stellar line or band-head at λ4752 can be attributed to carbon. And yet one instinctively feels that carbon in some form or other is the real origin. On this point there is room for further research on carbon spectra under varying conditions. It may be, of course, that such a line or band occurs in some of the already existing photographs of carbon spectra in the various institutions where spectroscopic research is carried on.

(Note 8.) It is rather curious that this band (head at λ4848·4) has only been noted in ρ Persei of the M type spectra studied by Stebbins. It is apparently identical with one of the weaker TiO_2 bands, but several other bands of TiO_2 of the same intensity are found in o Ceti and other M type stars.

References.

(*a*) Hale's sunspot maps.
(*b*) Rowland's solar tables.
(*c*) *Lick Obs. Bull.*, 2, 78.
(*d*) *Pub. Ast. Obs. Univ. Michigan*, 2, 103.
(*e*) *Decennial Publications, Univ. Chicago*, 8, 253.
(*f*) *Ast. Phys. Journ.*, 56, 457.
(*g*) *M.N., R.A.S.*, 72, 548.

* *Pub. Michigan Obs.*, 2. Plate following p. 102.

The Spectrum of γ Cassiopeiæ, H_β *to* B. *Stonyhurst College Observatory.* (Plate 16.)

(*Communicated by the Director.*)

§ 1. *Introduction.*—The present paper may be regarded as complementary to the discussion of the spectrum of γ Cassiopeiæ, published by the late Father Sidgreaves in May 1899.* That paper embraced a study of a series of plates of the spectrum distributed over the preceding eight years. A list of wave-lengths of bright and dark lines was given, from λ3963 to λ5576, but, except in a few cases, origins were not ascribed to the lines. The paper was illustrated by a drawing of the spectrum from a photograph taken on 1898 March 7. The plates had been obtained with the 4-inch Thorp objective-prism, and with the Hilger direct-vision prism slitless spectroscope, attached first to the 8-inch, and afterwards to the 15-inch equatorial. The present communication regards plates secured with the Hilger spectroscope and 15-inch equatorial, but with different settings from those employed by Father Sidgreaves. This has necessitated the construction of an independent wave-length curve. The plates cover the years 1921–22. As the wave-lengths of the lines determined by Father Sidgreaves have been reprinted in the exhaustive study of the spectrum of the star made by Dr. Curtiss, † the lines of shorter wave-lengths have not been remeasured on the plates taken in the later years. We have contented ourselves with measuring from H_β to the broad atmospheric band B, which gives a sufficient overlap of wave-lengths of lines for the purpose of comparison with those determined by Father Sidgreaves. All the later photographs were taken on Ilford Panchromatic Plates, the length of the spectrum from H_β to B measuring nearly 8 mm. The plates were measured to the one-hundredth of a mm., and in the red end of the spectrum, from λ6550 to λ6890, one unit of measurement is equivalent to about 5 Ångström units. This fairly represents the probable error of the wave-lengths of the lines in that region of the spectrum.

§ 2. *The Wave-Lengths.*—Of about twenty plates taken the best six, which had the clearest definition and the greatest extention of the spectrum towards the red end, were selected. These were carefully measured by the Rev. H. Macklin, who is responsible for both the measures and for the construction of the wave-length table. Each plate was measured independently, both in the direct and the reversed positions. Any line not detected when the plate was reversed was rejected. Only those lines were measured which could be traced right across the spectrum, as, on account of the extreme faintness of the majority of the lines, there would otherwise have been a possibility of measuring spurious and not real lines. The twofold identification of each line on each plate, due to reversing the plate, also helped to keep the measurements confined to genuine lines.

For the purpose of identifying the lines, the formula $M\lambda + aM + b\lambda + c = 0$ was adopted, where λ is the wave-length, M the micrometer

* *Monthly Notices R.A.S.*, 59, 505.
† *Publ. Detroit Obs.*, 2, 11, 1916.

measure of a line in the spectrum, and *a*, *b*, and *c*, constants. These constants were determined from six known lines on plates of spectra of other stars, taken under the same conditions, which lines were carefully measured by three independent observers.

The resulting value for λ was $2772{\cdot}2 + \frac{3246503}{9509{\cdot}3 - M}$, a value which gave very satisfactory results in the identification of lines. Broad bands, however, such as the very bright H_α line, and the absorption B band, gave larger errors towards the red end of the spectrum, due possibly to photographic halation, narrow lines in the same region not showing the same differences on the individual plates. As a further test of the accuracy with which the wave-lengths of the lines in the red end of the spectrum could be obtained, where the dispersion is so very small, the positions of the more prominent lines between D_3 and H_α were measured independently by Father Cortie on two photographs obtained by Father Sidgreaves in 1908, on Wratten & Wainwright Panchromatic Plates, with a different setting of the spectroscope, and accordingly with a different wave-length curve. Three settings of the micrometer wire were made on each line, both in the direct and in the reverse positions, and a value of the wave-length was derived from the mean of the six readings. The following is the result of the comparison:—

Macklin.	Cortie.	Macklin.	Cortie.
5924, 5965 } 5944	5950	6132	6135
		6140, 6158 } 6149	6148
5984	5989		
6031	6029	6211 *a*	6218 *e*
6057	6057	6255	6258
6082	6075	6292	6291
6098	6099	6442	6446

With the exception of one instance, where the lines measured may not have been identical, the results are fairly accordant, and the differences are of the order represented by 0·01 mm. in this part of the scale. In the case of line 6211 evidently one observer was measuring the bright edge of an absorption line.

§ 3. *The Table.*—In the following table the first column contains the wave-lengths of lines measured on more than two of the six plates. Lines measured on one or two plates only are called probable lines, and constitute the second column. The intensity scale in the third column depends purely on the darkness or brightness of the line, and not on its width, zero representing a line just discernible on the continuous spectrum, and 10 that of the bright H_β line. H_α constitutes a class by itself, and should be at least 20 on the chosen scale. The fourth column gives the character of the line absorption or emission. In the fifth column are given the wave-lengths of the lines determined by Father Sidgreaves in the portion of the spectrum where the measures overlap, and in the red end the wave-lengths given by Miss Mary Lea Heger.* Similarly, the

* *Lick Obs. Bull.*, No. 337, x. 148.

sixth column sets forth the wave-lengths of the lines determined by Dr. Curtiss.* The observations of Miss Heger were made with a 3-prism spectrograph, and those of Dr. Curtiss with a single-prism spectrograph. The seventh column contains the probable origins of the lines. Unless otherwise stated in the column for remarks they are all spark lines of the various elements, and they are practically all chromospheric lines. In preparing this column the authorities consulted were the tables of Exner and Haschek and the useful tables of Twyman, for the chromosphere the table of Mitchell, and for the flash spectrum that of Adams and Burwell.

Wave-lengths.		Intensity.	Character.	Sidgreaves.	Curtiss.	Probable Origin.	Remarks.
Certain Lines.	Probable Lines.						
4862		10·0	*e*	4861	4861·5	H	H_β.
76		1·2	*e*			75·7 V	
	4884	2·0	*e*			83·6 V	
93		1·0	*e*			91·7 Fe	
4911		1·3	*e*			11·3 Ti	Diffuse line.
	4913	1·5	*a*			13·8 Ti	
24		0·9	*e*		4925·7	24·1 Ti	4921·9 He.
	32	0·0	*e*			34·3 Ba	
	39	1·0	*e*			39·4 Fe	
51		1·4	*e*			50·3 Fe	
	59	0·3	*e*			57·8 Ti	Diffuse line.
	67	1·0	*e*			66·3 Fe	
85		0·7	*e*			85·4 Fe	
	92	1·0	*e*			91·2 Ti	
5000		1·2	*e*			00·5 Ni	Arc line.
	5006	1·5	*e*	5006		06·3 Fe	
17		2·3	*e*			15·7 He	Diffuse line.
	21	0·0	*e*	20	5018·3	20·1 Ti	Arc line. Chromospher
					18·1		
	33	1·3	*e*			33·0 Ni	
	40	1·0	*a*			40·1 Ti	
				48 *a*	48 *a*	47·7 He	
58		1·0	*e*			58·0 ?	Chromosphere.
	69	1·5	*a*			68·9 Fe	
	75	1·3	*e*			74·9 Fe	
	5082	0·0	*e*			81·3 Ni	Arc line. Chromospher

* *Publ. Detroit Obs.*, 2, 13, 14, 1916.

Wave-lengths. Certain Lines.	Wave-lengths. Probable Lines.	Intensity.	Character.	Sidgreaves.	Curtiss.	Probable Origin.	Remarks.
99		1·0	*e*			99·4 Ni	Arc line. Chromosphere.
				5104 *a*	5106 *a*		
	5116	0·0	*e*			15·6 Ni	Arc line. Chromosphere.
	25	0·5	*e*			25·3 Fe	
	31	0·3	*e*			31·7 Fe	Arc line. Chromosphere.
	42	1·5	*e*			42·7 Ni	,, ,,
				60 *a*	62		
67		1·8	*e*		68 *a*	67·5 Mg	b_4.
				70 *e*	69 *e*	69·0 Mg	b_3.
	73	1·0	*e*		74 *a*	72·8 Mg	b_2.
					80 *a*		
					86 *e*	83·8 Mg	b_1.
				5214 *a*	5214 *e*		
32		1·5	*e*		34 *e*	33·1 Fe	Diffuse line.
57		1·0	*e*	56		57·1 Sr	
80		1·3	*e*		78	80·6 Co	Arc line. Chromosphere.
					81 *a*		
					86 *e*		
	5296	0·0	*e*	95 *a*	94 *a*	96·9 Cr	
98		1·7	*a*			98·4 Cr	Diffuse line.
315		2·2	*e*	16	17	15·2 Fe	Arc line. Chromosphere. Diffuse line.
	5328	1·0	*e*		27	28·2 Fe	
	38	1·3	*e*			37·9 Cr	
				50 *a*	55 *a*		
61		1·0	*e*			61 8 Fe	Arc line. Chromosphere.
374		0·9	*e*	5376 *a*	5376 *a*	73·8 Fe	,, ,,
	5394	1·0	*e*			93·7 Ce	
	5405	1·0	*e*			04·3 Fe	
				5411 *a*			
427		0·6	*e*		5426	27·4 ?	Chromosphere.
	48	0·0	*e*			47·1 Fe	
	57	1·0	*e*			55·8 Fe	
77		0·5	*e*			77·1 Ni	
	93	1·0	*e*			92·1 Fe	Arc line. Chromosphere.
	5507	1·3	*e*			07·0 Fe	
				5524 *a*	5523 *a*		
535		1·1	*e*	40 *e*	37 *e*	35·6 Fe	Arc line. Chromosphere.

Wave-lengths. Certain Lines.	Wave-lengths. Probable Lines.	Intensity.	Character.	Sidgreaves and Heger.	Curtiss.	Probable Origin.	Remarks.
66		1·1	*e*			65·7 Ti	
	73	1·5	*a*	76 *e*		73·1 Fe	
85		1·2	*e*			85·3 ?	Chromosphere. 84·7 Al line V.
	97	1·0	*e*			97·7 ?	Chromosphere.
	5625	1·0	*e*			25·1 V	
	40	0·0	*e*			40·5 Er	Arc line. Chromosphere
5655		1·7	*e*			55·6 Fe	
	70	1·0	*e*			70·1 Ni	Arc line. Chromosphere
	91	1·0	*e*			90·7 Si ?	Chromosphere.
	5704	1·0	*e*			03·8 V	
	12	0·0	*e*			12·1 Fe	Arc line. Chromosphere
	28	1·0	*e*			27·3 V	
	64	0·5	*e*		5760 *a*	63·1 Fe	
	97	1·5	*e*			97·8 Ti	Arc line. Chromosphere
5808		0·8	*e*			06·9 Fe	,, ,,
	5827	1·0	*e*			27·8 V	Arc line. Chromosphere 28·1.
	50	0·5	*e*			?	
	63	1·0	*e*	5862 *a*	5862 *a*	62·8 Fe	
5880		1·3	*e*	76·1	76	75·6 He	D_3.
					82 *a*		
				90·0 *a*			D_2.
				93·0 *a*			
	5903	1·0	*e*	95·9 *a*			D_1.
	13	0·5	*e*			14·2 Fe	
	26	0·0	*e*			24·9 V	
	50	1·0	*e*	5953		52·9 Fe	Arc line. Chromosphere
5942		0·8	*e*			42·0 Ti	
	65	1·3	*e*			65·9 Ti	
84		0·5	*e*			83·9 Fe	
	6010	0·0	*e*			08·9 Fe	
6030		0·0	*e*			27·3 Fe	
45		1·4	*e*			43·6 Ce	Arc line. Chromosphere Diffuse line.
57		0·0	*e*			56·2 Fe	
	82	1·0	*e*			81·7 V	
98		0·8	*e*			98·9 Ti	

Wave-Lengths. Certain Lines.	Wave-Lengths. Probable Lines.	Intensity.	Character.	Heger.	Curtiss.	Probable Origin.	Remarks.
	6125	1·0	*e*			25·2 Ni?	Chromosphere.
	32	1·0	*a*			32·2 Y	
	40	1·0	*e*			41·9 Ba	Arc line. Chromosphere.
	49			6148	6149		Curtiss diffuse line. Mean of 6140, 6158.
	58	1·5	*e*			57·9 Fe	6160 *e*. Sherman.
4172		1·0	*e*			73·4 Fe	Diffuse line.
	81	1·5	*e*			80·4 Fe?	
	6192	1·0	*e*			91·8 Fe	
1211		1·6	*a*			10·9 Sc	Arc line. Chromosphere.
				6236		32·9 Fe	Probable line.
				6247		46·5 Fe	
2255		1·8	*e*			56·6 Ni	Arc line. Chromosphere. Diffuse line.
				6273	6278		
6292		0·8	*e*			91·3 Fe	Arc line. Chromosphere.
96		2·6	*a*		6307 *a*		*a* band. Atmospheric.
	6315	1·5	*e*	6318	6320	16·2 Fe	Arc line. Chromosphere.
6335		1·0	*e*		6340 *a*	35·5 Fe	
	45	1·0	*e*	45	50	47·1 Si	
	52	1·0	*e*		50 *e*	55·0 Fe	6356 Sherman. Arc line. Chromosphere.
				70 *e*	67 *a*	71·4 Si	
89				84		90·7 La	
				6418		17·1 Fe	
				31		31·3 Fe	
6442		0·3	*e*			39·5 Ca	
56				56		56·7 Fe	6456 mean of 6442 and 6470.
70		1·3	*e*			71·9 Ca	
				92		91·8 Ti	
6513		1·0	*e*	6514		16·3 Fe?	
63		20·0	*e*	63	6563	63·6 H	H_α. A broad line.
6646		1·3	*e*			44·9 Ti	Arc line. Diffuse line. Chromosphere 6640?
6716		0·6	*e*			17·0 Ti	Chromosphere 6715. Diffuse line.
69		0·7	*e*			68·0 Ni	Arc line.
	6855	1·8	*e*			55·5 Fe	Arc line.
6889		5·1	*e*				B band. Atmospheric.

§ 4. *The Variability of the Spectrum.*—A comparison of the plates, taken in this part of the spectrum in 1908, with those photographed in 1921, does not show any variability in the spectrum, as far as the prominent lines of hydrogen and the faint bright line D_3 are concerned. In those cases, too, in which different observers have assigned different character marks, as emission or absorption, to the same lines, although the possibility of variability in the spectrum must not be altogether excluded, it is much more likely that the lines have both emission and absorption components.

§ 5. *The Character of the Spectrum.*—The predominant character of the spectrum is the series of bright hydrogen lines, decreasing in intensity with decrease in wave-length. The lines H_α and H_β with the dispersion employed are very broad, extending over 20 and 8 Ångström units respectively, even allowing that the great breadth of H_α may be partly due to photographic halation. They are probably double lines, unresolved on account of the smallness of the dispersion. They are bounded on either edge by absorption bands, those about H_α extending over 50 and those about H_β over 12 Ångström units, on either side of the bright lines. Otherwise the spectrum consists mainly of bright lines of no great intensity, interspersed with absorption bands, the bright lines being much the more numerous in this portion of the spectrum. These constitute a spectrum greatly resembling that of the sun's chromosphere. The helium lines in this part of the spectrum are collated with the lines in the star as follows:—

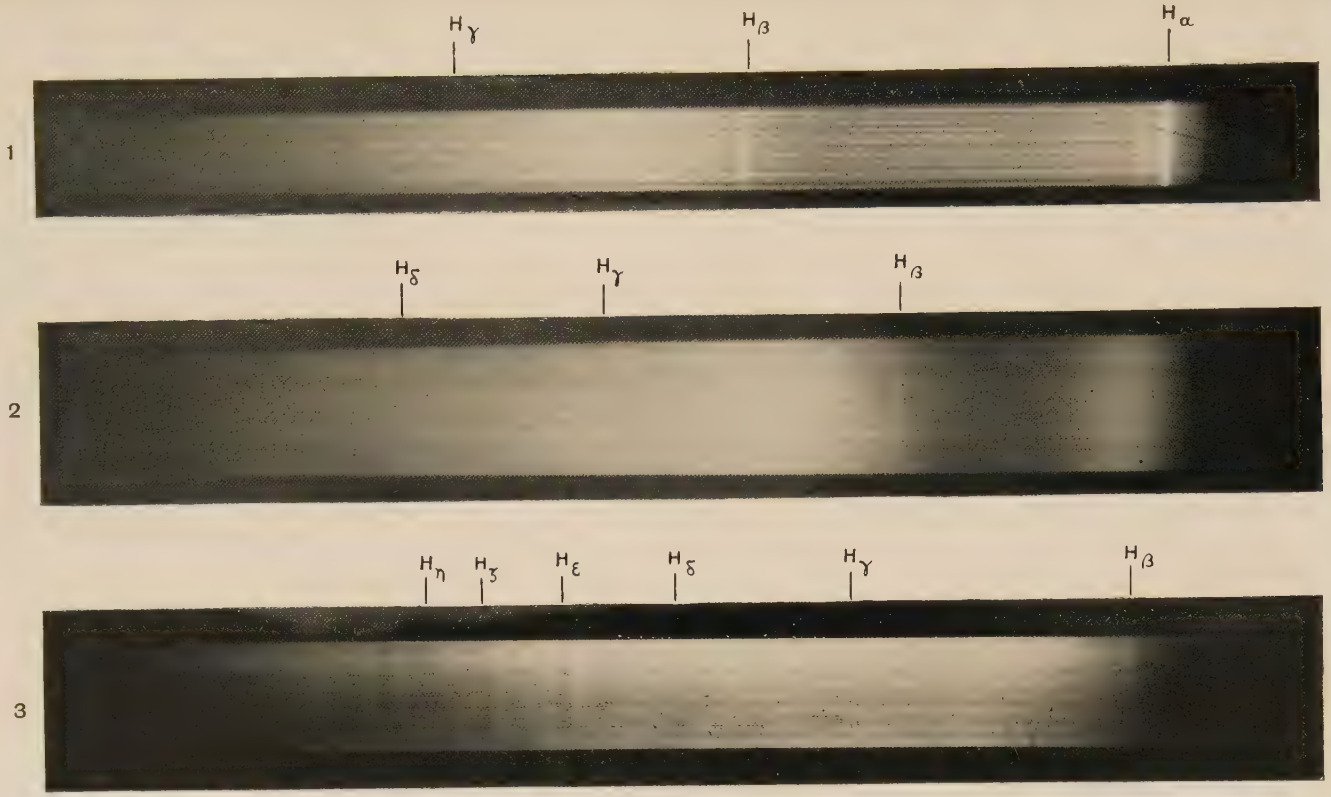
Hélium.	γ Cassiopeiæ.	
λ.	λ.	Intensity.
4922	4924 *e*	0·9
5016	5017 *e*	2·3
5048	5048 *a*	2·0
5876	5880 *e*	1·3 D_3
6678		

The line 5048 absorption is given by Father Sidgreaves, who also recorded all the other lines of the helium spectrum from λ4026 to λ4713, as matched by absorption lines in the spectrum of the star. Here again the difference of interpretation would point to the presence of both emission and absorption lines. And, if the accuracy of the wave-lengths could be trusted to that extent, the bright lines would be to the less refrangible side of the dark lines, as in the spectrum of new stars.

Description of the Plate.

1. Spectrum taken with Hilger direct-vision slitless spectroscope on 15-inch equatorial, by Father Rowland, 1921 April 2. Panchromatic plate.
2. Spectrum taken with same instruments by Father Sidgreaves, 1898 April 7. Isochromatic plate.
3. Spectrum taken with 4-inch Cooke-Thorp objective-prism camera by Father Sidgreaves, 1905 May 5. Ordinary plate.

Stonyhurst College Observatory:
1924 *June* 9.



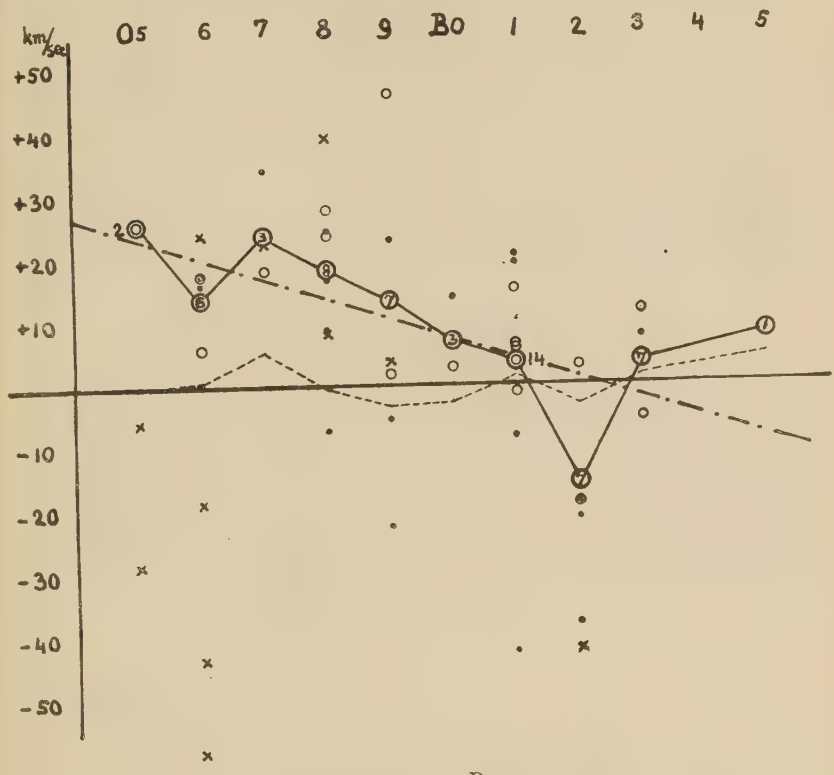
Complete Spectrum of γ Cassiopeiæ.

The Stationary Calcium Lines in O- and B-type Stars.

By H. Kienle.

(*Communicated by the Secretaries.*)

In *M.N.*, **84**, 80, Professor J. S. Plaskett has discussed the radial velocities of O- and early B-type stars as derived from ordinary star



Explanation of Diagram.

Abscissæ: Spectral type.
Ordinates: differences of radial velocities.
● = $V_O - V_{Ca}$ of weight 1.
◯ = ,, ,, 2.
(*n*) = ,, mean of weight *n*.
× = ,, stars with bright lines.
— - — = representation of formula $V_O - V_{Ca} = +7{\cdot}0 - 3{\cdot}24$ (Sp. – Bo).
- - - - = mean values of $V_{Ca} - V_\odot$.

lines and from the sharp lines H and K of Ca^+. He found that there are real differences between both velocities. Looking nearer we shall find that, though there are some great negative values, the overwhelm-

ing number of differences $V_o - V_{Ca}$ (the indices o and Ca refer to star lines and calcium lines) are positive. A more detailed paper on this subject will appear shortly. Part of the results may be communicated here.

1. The radial velocities from the Ca^+-lines (V_{Ca}) are in full accordance with the corresponding components of the solar motion ($V_\odot$). In the mean we have:

$$V_{Ca} - V_\odot = -0\cdot4 \pm 0\cdot5 \text{ (p.e.) km./sec.}$$
$$\text{p.e. of one difference} \pm 4\cdot1 \text{ km./sec.}$$

2. For stars with bright lines (O5e to B2e) there is in the mean agreement between the radial velocities from star and from calcium lines. But the dispersion of individual values is great and the average absolute radial velocity is exactly the same as for planetary nebulæ (30 km./sec.).

3. If the stars with absorption lines are arranged in the order of spectral types, and stars of Young's list are added, we find a close connection between spectral type and red-displacement of stellar lines. We have the linear correlation:

$$V_o - V_{Ca} = +7\cdot0 - 3\cdot24 \text{ (Sp. – Bo)}$$
$$\pm 1\cdot6 \pm 0\cdot68 \text{ (p.e.).}$$

p.e. of one difference of weight unity $= \pm 12\cdot6$ km./sec. The unit of spectral type (Sp.) = one-tenth of Harvard class.

In the diagram we have represented these facts. We see that on the negative side there are almost exclusively stars with bright lines (which evidently behave abnormally) or observations of small weight. If interpreted as gravitational displacements, the differences $V_o - V_{Ca}$ would require either great mass (m) or great mean density (δ) of these early-type stars, or both, as may be seen from the following table, where m and δ are expressed in units of the same quantities for the sun.*

Type.	$\sqrt[3]{m^2\delta}$.	δ_{100}.	m_1.
B1	6·3	0·02	16
O9	17·1	0·50	71
O7	27·9	2·2	147
O5	38·7	5·8	241

δ_{100} = mean density for $m = 100\odot$; m_1 = mass for mean density $\delta = 1\odot$.

Göttingen:
1924 *May* 12.

* We do not know whether this is the real explanation of the observed displacements; but we may also say that we cannot believe that we have to deal with real relative motions of stars and calcium clouds.

Concerning Ephemerides of Small Planets. By T. C. Hudson.

This paper deals with a somewhat *general* problem of small perturbations, that is to say, of slow departures in regard to directivity, and small departures in regard to magnitude or distance, from a state of simple elliptical orbital motion in a plane about a focus.

With particular causes of perturbation it does not deal; neither, therefore, does it deal with numerical examples; but although the theory so far as it goes is algebraical, it lends itself of course to the consideration of any causes and any numerical examples.

For convenience the paper may be divided into three parts:—

1. Methods of analysis and synthesis;
2. Application to simple orbit;
3. Extension to perturbed orbit.

Part 1 consists of a calculus of directivities and a calculus of journeys. Both are easy, and they merge more naturally than they divide, and they will be treated together as one calculus. Its elements are not new. The logical developments from them are capable of simple statement. Application is potent, and with few exceptions verification is easy.

In order to prevent confusion with other established methods or notations that for one reason or another have not lent themselves well to this particular problem, it has seemed advisable to avoid using words and symbols likely to convey old meanings and to suggest old associations rather than new. And it is hoped that the nature of a few novelties as to name or as to symbol will be readily understood from their context.

1. *Methods of Analysis and Synthesis.*—Let us think of the following table of similarities, with the idea of three opposed pairs of directivities mutually at right angles:—

Table I.

Particular case	Up, down.	North, south.	East, west.
General symbols	α , $\acute{\alpha}$	η , $\acute{\eta}$	ω , $\acute{\omega}$
Suggested pronunciation	ah, ha	ee , he	owe, hoe

In order to make an algebra in which "addition" and "multiplication" shall be consistent with the facts of plane and solid geometry, it has been found (for many practical purposes) to be necessary and sufficient to agree to certain conventions as to "multiplication" products, as in Tables II., III., and IV.

Table II.

Particular case	"*Half* (a league) *onward.*"	
Qualities	Magnitude.	Directivity.
General symbols	A	α
	H	η
	. . .	. . .

$$A\alpha = |\,A\,|\,\alpha = A = \alpha\,|\,A\,| = \alpha A.$$

TABLE III.

Scalar Products.

	α	η	ω
α	1	.	.
η	.	1	.
ω	.	.	1

E.g., $\alpha^2 = \ldots = \ldots = 1$; $\eta\omega = \ldots = \ldots = 0$.

TABLE IV.

*Directed Products: Two Kinds (v and V), opposed.**

(v)	α	η	ω	
α	.	ω	$\acute{\eta}$	α
η	$\acute{\omega}$	.	α	η
ω	η	$\acute{\alpha}$	.	ω
	α	η	ω	(V)

E.g., $v\alpha\alpha = 0 = V\alpha\alpha$; $v\eta\omega = \alpha = V\omega\eta$.

Journeys: their "Addition" and "Multiplication."

"From station 0 to station 1 and then from 1 to 2" is the same for many purposes as "from 0 to 2." In this sense we write

$$A + B = C$$

and (assuming the property of parallelism)

$$B + A = C.$$

We then find that if we multiply, the results are true to nature;

$$A^2 + B^2 + 2AB = C^2$$

is true to geometry; $\alpha\beta$ is the cosine of the angle between the directivities α and β.

Also we find that $vAB = vAC = vCB =$ twice the directed area of the triangle of journeys, the directivity being one or other of the two that are at right angles to the plane.

* Suffix V is inconvenient; hence the ν in $\tau_v\tau_\nu$.

Also that

$$\alpha \cdot \alpha + \eta \cdot \eta + \omega \cdot \omega = 1.$$

$\alpha \cdot \alpha$ means "alpha-ward part of."
$\eta \cdot \eta$ means "eta-ward part of."
$\omega \cdot \omega$ means "omega-ward part of."

$\alpha \cdot \alpha + \eta \cdot \eta + \omega \cdot \omega$ means "all of."

And

$$\upsilon\alpha = \eta \cdot \omega + \dot{\omega} \cdot \eta = \eta \cdot \omega + \omega \cdot \dot{\eta}.$$
$$V\alpha = \dot{\eta} \cdot \omega + \omega \cdot \eta = \eta \cdot \dot{\omega} + \omega \cdot \eta.$$
$$\upsilon\alpha V\alpha = V\alpha\upsilon\alpha = \eta \cdot \eta + \omega \cdot \omega = \text{"}non\text{-alpha-ward part of."}$$
$$\alpha \cdot \alpha + \upsilon\alpha V\alpha = 1 = \text{"all of."}$$

"Part" means journey, not undirected projection.

Also

$$\alpha\upsilon\eta\omega = 1\,; \quad \alpha\upsilon\eta\alpha = 0, \text{ etc.},$$

and, for any three directed lines LMN, at whatever angles, either LυMN or else LVMN is (the positive measure of the volume of) a box.

Again, for any three directed lines,

$$VA\,VBC + C \cdot AB = B \cdot AC$$

and

$$VA\,VBC + VB\,VCA + VC\,VAB = 0.$$

Also

$$VAB\,VCD + AD \cdot BC = AC \cdot BD.$$

And if N be an integer and

$$N! = 1 \cdot 2 \cdot 3 \cdot \ldots N$$

and

$$\sin x + x^3/3! + x^7/7! + \ldots$$
$$= x + x^5/5! + x^9/9! + \ldots$$

and

$$1 + x^4/4! + x^8/8! + \ldots$$
$$= \cos x + x^2/2! + x^6/6! + \ldots$$

then

$$\alpha \sin A = \sin(\alpha A) = \sin \mathrm{A}\,;$$ the *directed* Sine; S.

but

$$\cos A = \cos(\alpha A) = \cos \mathrm{A}\,;$$ the undirected cosine; C.

[Mnemonics, Sine and Tangent; *C*osine.]

Also

$$TC = S.$$

If the symbol ϵ denote the blank form of operator

$$1 + [\quad](1 + \tfrac{1}{2}[\quad](1 + \tfrac{1}{3}[\quad](1 + \tfrac{1}{4}[\quad](\ldots)$$

and

$$\sin(\alpha\gamma) = \mathrm{S},\ \tan(\alpha\gamma) = \mathrm{T}\,;\ \cos(\alpha\gamma) = C$$
$$\sin\gamma = S,\ \tan\gamma = T,\ \cos\gamma = C$$

and $$\tau_v = \epsilon v a(\gamma + \gamma),$$

then $$\tau_v = \tau_v(\alpha \cdot \alpha + \eta \cdot \eta + \omega \cdot \omega)$$
$$= \ldots$$
$$= 1 + 2C^2(1 + v\mathrm{T})v\mathrm{T},$$

whereas
$$\tau_\nu = 1 + 2C^2(1 + V\Gamma)V\mathrm{T},$$

in which
$$\tau_v\tau_\nu = 1 = \tau_\nu\tau_v.$$

τ_v and τ_ν are the equal and opposed operators of turn, about the axis whose directivities are α and $\acute{\alpha}$, through the dihedral angle whose circular measure is 2γ.

If the dot be used to denote differentiation with respect to any variable, say γ, and
$$\dot{\mathrm{I}}_v = (1 + v\mathrm{T})\dot{\mathrm{T}}2C^2$$

and
$$\dot{\mathrm{I}}_\nu = (1 + V\mathrm{T})\dot{\mathrm{T}}2C^2$$

and as before
$$\tau_v = 1 + (1 + v\mathrm{T})v\mathrm{T}2C^2$$

and
$$\tau_\nu = 1 + (1 + V\mathrm{T})V\mathrm{T}2C^2,$$

then we find that
$$\dot{\tau}_v\tau_\nu = v\dot{\mathrm{I}}_v \qquad \dot{\tau}_\nu\tau_v = V\mathrm{I}_\nu$$
$$\tau_v\dot{\tau}_\nu = V\dot{\mathrm{I}}_v \qquad \tau_\nu\dot{\tau}_v = v\mathrm{I}_\nu$$

$$\tau_v\dot{\mathrm{I}}_\nu = \mathrm{I}_v \qquad \tau_\nu\dot{\mathrm{I}}_v = \mathrm{I}_\nu$$
also $\tau_v\ddot{\mathrm{I}}_\nu = \ddot{\mathrm{I}}_v$ and $\tau_\nu\ddot{\mathrm{I}}_v = \ddot{\mathrm{I}}_\nu.$

Again, if
$$\mathrm{H} = V\mathrm{A}d/dt\mathrm{A},$$

then
$$\frac{d}{dt}\mathrm{H} = \frac{d}{dt}(V\mathrm{A}d/dt\mathrm{A})$$
$$= V\frac{d}{dt}\mathrm{A}\,\frac{d}{dt}\mathrm{A} + V\mathrm{A}\left(\frac{d}{dt}\right)^2\mathrm{A} = 0 + V\mathrm{A}\left(\frac{d}{dt}\right)^2\mathrm{A}.$$

Hence, if $V\mathrm{A}\left(\frac{d}{dt}\right)^2\mathrm{A} = 0$, then $V\mathrm{A}\frac{d}{dt}\mathrm{A}$ is invariable both in respect of its directivity $Va\frac{da}{|da|}$ and in respect of its magnitude $A^2\frac{|da|}{dt}$

Also, if
$$\mathrm{H} = V\mathrm{A}d/dt\mathrm{A},$$

then
$$V\mathrm{H}\mathrm{A}A^{-3} = va\,Vad/dta = d/dta.$$

2. *Simple Orbit.*—If the directed line from fixed focus to moving point be
$$\mathrm{R}, = \rho R,$$

and $$\rho + \dot{\rho} = 0, \quad \text{and} \quad RP = 1,$$

and $$^1\mathrm{R}dt = d\mathrm{R} \quad \text{and} \quad ^2\mathrm{R}dt = d\,^1\mathrm{R},$$

then the directed velocity is represented by $^1\mathrm{R}$ and the directed acceleration by $^2\mathrm{R}$, and the law of directed force as well as the "three equations of motion" are in the single (*directed*) equation

$$^2\mathrm{R}R^2 = \dot{\rho}.$$

Operating with $V\mathrm{R}$, integrating, and writing A for $V\mathrm{R}\,^1\mathrm{R}$, we find :— A is invariable, α is immovable, and A is constant.

Operating with $\upsilon\mathrm{A}$, we find

$$\upsilon\mathrm{A}\,^1\mathrm{R} = \rho + \mathrm{E},$$

in which $\mathrm{AE} = 0$, and E is invariable.

A defines the planarity and (twice) the variation of area.

E defines pericentron and the eccentricity.

Among scalar results are :

$$A^2 = \mathrm{R}\upsilon\mathrm{A}\,^1\mathrm{R} = \mathrm{R}(\rho + \mathrm{E}) = R(1 + E\cos\theta)$$
(the polar equation of the conic)

$$\tfrac{1}{2}M(^1\mathrm{R})^2 = \tfrac{1}{2}M(1 + E^2 + 2E\cos\theta)/A^2$$
(an energy equation)

$$^2\mathrm{R}\mathrm{R} + P = 0$$ (virial)

$$|\,d\rho\,|/dt = (1 + E\cos\theta)^2/A^3\,; \quad |\,d\rho\,| = d\theta$$
(increment of longitude)

and an interesting identity of operations :

$$1 + R^3(d/dt)^2 = (1 + (d/d\theta)^2)([\quad][1 + E\cos\theta]).$$

3. *Perturbations.*—The directed equation of motion may be written

$$(1 + R^3(d/dt)^2)\mathrm{R} = \mathrm{F}.$$

Let

$$\mathrm{H} = V\mathrm{R}\,^1\mathrm{R}, \quad ^1\mathrm{R}dt = d\mathrm{R}, \quad ^2\mathrm{R}dt = d\,^1\mathrm{R}.$$

Then

$$^1\mathrm{H} = V\mathrm{R}^2\mathrm{R} = V\mathrm{R}\mathrm{F}R^{-3}$$
$$V\mathrm{H}\,^1\mathrm{H} = r\,.\,\mathrm{H}\mathrm{F}\,.\,R^{-2} \qquad *$$
$$\mathrm{H}\,^1\mathrm{H} = \mathrm{F}\,^1r$$
$$H^2 = \mathrm{H}^2 = K^2 + 2\int \mathrm{F}dr$$

in which K^2 is a constant which in general depends on the epoch chosen.

Let

$$\tau_\upsilon = \epsilon\upsilon\alpha(\gamma + \gamma), \qquad \tau_\nu\tau_\upsilon = 1 = \tau_\upsilon\tau_\nu,$$
$$C = \cos\gamma, \qquad \mathrm{T} = \alpha\tan\gamma$$

$$d\mathrm{I}_\upsilon = (1 + \upsilon\mathrm{T})d\mathrm{T}\,.\,2C^2$$

and $$d\mathrm{I}_\nu = (1 + V\mathrm{T})d\mathrm{T}\,.\,2C^2.$$

---

* Here $h\ r\ k\ p\ q$, $\hbar_0$, etc., are *directivity*, not number.

Then
$$\tau_v = 1 + v\mathrm{T}(1 + v\mathrm{T}) \,.\, 2C^2$$
$$\tau_\nu = 1 + V\mathrm{T}(1 + V\mathrm{T}) \,.\, 2C^2$$
$${}^1\tau_v\tau_\nu = v\,{}^1\mathrm{I}_v \qquad {}^1\tau_\nu\tau_v = V\,{}^1\mathrm{I}_\nu$$
$$\tau_v\,{}^1\tau_\nu = V\,{}^1\mathrm{I}_v \qquad \tau_\nu\,{}^1\tau_v = v\,{}^1\mathrm{I}_\nu$$
$$\tau_v\,{}^1\mathrm{I}_\nu = {}^1\mathrm{I}_v \qquad \tau_\nu\,{}^1\mathrm{I}_v = {}^1\mathrm{I}_\nu$$
$$\tau_v\,{}^2\mathrm{I}_\nu = {}^2\mathrm{I}_v \quad \text{and} \quad \tau_\nu\,{}^2\mathrm{I}_v = {}^2\mathrm{I}_\nu.$$

Now let
$$\tau_v h = h_0, \qquad dh_0 = 0,$$
and
$$\tau_v r = r_0.$$
Then
$${}^1h = {}^1\tau_\nu h_0 = {}^1\tau_\nu\tau_v h = V\,{}^1\mathrm{I}_\nu h,$$
whence
$$r \,.\, \mathrm{HF}H^{-2}R^{-2} = H^{-2}V\mathrm{H}\,{}^1\mathrm{H} = Vh\,{}^1h = Vhvh\,{}^1\mathrm{I}_\nu.$$

Now let
$$h\,{}^1\mathrm{I}_\nu = 0,$$
so that
$${}^1\mathrm{I}_\nu = Vhvh\,{}^1\mathrm{I}_\nu = r \,.\, \mathrm{HF}H^{-2}R^{-2}$$
and
$$V\,{}^1\mathrm{I}_\nu\mathrm{R} = 0.$$

Again, if $R_0 = R$,
$$\mathrm{R} = \tau_\nu\mathrm{R}_0$$
$${}^1\mathrm{R} = {}^1\tau_\nu\mathrm{R}_0 + \tau_\nu\,{}^1\mathrm{R}_0$$
$$= V\,{}^1\mathrm{I}_\nu\mathrm{R} + \tau_\nu\,{}^1\mathrm{R}_0$$
$$= \tau_\nu\,{}^1\mathrm{R}_0$$
$${}^2\mathrm{R} = V\,{}^1\mathrm{I}_\nu\tau_\nu\,{}^1\mathrm{R}_0 + \tau_\nu\,{}^2\mathrm{R}_0 = \tau_\nu\,{}^2\mathrm{R}_0 + Vr\,{}^1\mathrm{R}(\mathrm{HF}H^{-2}R^{-2})$$
$$= \tau_\nu\,{}^2\mathrm{R}_0 + h \,.\, h\mathrm{F}R^{-3}.$$

Writing F_0 for $\tau_v\mathrm{F}$ we have
$$\mathrm{F}_0 = \tau_v\mathrm{F} = \tau_v(\mathrm{R} + {}^2\mathrm{R}R_0{}^3)$$
$$= \mathrm{R}_0 + {}^2\mathrm{R}_0R_0{}^3 + h_0 \,.\, h_0\mathrm{F}_0,$$
whence
$$\mathrm{R}_0 + {}^2\mathrm{R}_0R_0{}^3 = vh_0Vh_0\mathrm{F}_0 = \mathrm{G}_0 \text{ say},$$
and the motion of R_0 is *planar*, dh_0 being $= 0$.

Now let
$$R_0 = P\epsilon\lambda$$
$$H_0\epsilon\mu = K \qquad (H_0 \text{ being } = H, \text{ and } dK = 0)$$
$$\epsilon\nu \,.\, dt = dt_0 \qquad (\text{this } \nu \text{ being of course } \textit{number})$$
$$K^2 = P(1 + E\cos\theta), \qquad dE = 0$$
$$Kdt_0 = P^2d\theta$$
$$h_0 = k$$
$$r_0 = \epsilon Vk\phi \,.\, p$$
and
$$p = \epsilon Vk\theta \,.\, p_0, \qquad dp_0 = 0, \qquad Vkp = q.$$

Then
$$1 + P^3\left(\frac{d}{dt_0}\right)^2 = \left(1 + (d/d\theta)^2\right)\left([\quad](1 + E\cos\theta)\right)$$
$$^1\mathrm{R}_0 = d/dt\,\mathrm{R}_0 = \epsilon\nu \,.\, d/dt_0\mathrm{R}_0$$
$$^2\mathrm{R}_0 = \epsilon 2\nu \,.\, \left(\left(\frac{d}{dt_0}\right)^2 + \frac{d\nu}{dt_0}\frac{d}{dt_0}\right)\mathrm{R}_0.$$
$$^2\mathrm{R}_0 R^3 = \epsilon(2\nu + 3\lambda) \,.\, \left\{-1 + \left(1 + P^3\left(\frac{d}{dt_0}\right)^2\right) + P^3\frac{d\nu}{dt_0}\frac{d}{dt_0}\right\}\mathrm{R}_0.$$

Hence, writing $dx = \dot{x}d\theta$, etc., we have

$$K^{-2}\epsilon(-4\lambda - 2\nu) \,.\, {}^2\mathrm{R}_0 R^3 = \ddot{r}_0 + \dot{r}_0(2\dot{\lambda} + \dot{\nu}) + r_0(1 - (1 + E\cos\theta)^{-1} + \dot{\nu}(\dot{\lambda} + E\sin\theta(1 + E\cos\theta)^{-1}) + \ddot{\lambda} + \dot{\lambda}^2)$$

whence

$$\mathrm{G}_0 K^{-2}\epsilon(-4\lambda - 2\nu) = r_0 + \ddot{r}_0 + r_0(\ddot{\lambda} + (\epsilon(-3\lambda - 2\nu) - 1)(1 + E\cos\theta)^{-1} + \dot{\lambda}^2 + \dot{\nu}(\dot{\lambda} + E\sin\theta(1 + E\cos\theta)^{-1})) + \dot{r}_0(2\dot{\lambda} + \dot{\nu}).$$

Now
$$k\epsilon(-2\lambda - \mu - \nu) = k\frac{P^2 d\theta \,.\, H_0 \,.\, dt}{R_0^2 d\theta \,.\, K \,.\, dt_0} = k\frac{H_0 dt}{R_0^2 d\theta} = \frac{\mathrm{H}_0 dt}{R_0^2 d\theta} = Vr_0\dot{r}_0,$$

hence
$$\dot{r}_0 = \nu r_0 Vr_0\dot{r}_0 = Vkr_0 \,.\, \epsilon(-2\lambda - \mu - \nu).$$

Also, since
$$r_0 = \epsilon Vk\phi \,.\, p \quad \text{and} \quad kp = 0,$$

we have
$$Vr_0\dot{r}_0 = k(1 + \dot{\phi}), \text{ as is otherwise obvious.}$$

Hence
$$|\dot{r}_0| = 1 + \dot{\phi} = \epsilon(-2\lambda - \mu - \nu),$$

so that ϕ is easily found if λ, μ, and ν are given. First, in regard to μ, we have
$$\epsilon(-2\mu) = 1 + 2\int \mathrm{F}dr K^{-2}.$$

Then in regard to λ we have

$$\mathrm{G}_0 r_0 K^{-2}\epsilon(-4\lambda - 2\nu) = 1 + r_0\ddot{r}_0 + \ddot{\lambda} + (\epsilon(-3\lambda - 2\nu) - 1)(1 + E\cos\theta)^{-1} + \dot{\lambda}^2 + \dot{\nu}(\dot{\lambda} + E\sin\theta(1 + E\cos\theta)^{-1})$$

in which
$$r_0\ddot{r}_0 = -\dot{r}_0{}^2 = -\epsilon(-4\lambda - 2\mu - 2\nu) = -1 + 4\lambda + 2\mu + 2\nu - \ldots$$

Hence to first order

$$\ddot{\lambda} + \lambda \frac{1 + 4E\cos\theta}{1 + E\cos\theta} = G_0 r_0 K^{-2} + 2\int G_0 dr_0 K^{-2} + \text{terms in } \nu,$$

[of which a solution was published in the *Proceedings of the London Mathematical Society*, 1923 February 8 :—

"The solution of the equation

$$d/d\theta[d^2\lambda/d\theta^2 + \lambda(1 + 4E\cos\theta)/(1 + E\cos\theta)] = d\xi/d\theta + 2\eta,$$

in which E is constant, may be written in the form

$$\begin{aligned}\lambda = {} & S(1 + EC)[\lambda_1 + \int C(1 + EC)^{-1}\xi d\theta] \\ & + C(1 + EC)\lambda_2/2E + C^2(2 + EC)(1 + EC)^{-1}\int \eta d\theta \\ & - C(1 + EC)^{-1}[\lambda_3 + \int\{C(2 + EC)\eta + S(1 + EC)\xi\}d\theta] \\ & + 2S(1 + EC)\int C(1 + EC)^{-3}[E\lambda_3 + \int\{(1 + EC)^2\eta + ES(1 + EC)\xi\}d\theta]d\theta,\end{aligned}$$

in which $C = \cos\theta$ and $S = \sin\theta$, and λ_1, λ_2, λ_3 are constants."]

Lastly, in regard to ν, Laplace showed that $\dot{\nu}$ contained no *secular* terms of the first and second orders as to perturbing masses, but the reference was not to small planets.

A Method of Extrapolation of Perturbations. By B. V. Noumerov.

(*Communicated by the Secretaries.*)

I have pointed out in my previous work * the advantage which is to be got in the numerical integration of the equations of perturbed motion by the introduction of special rectangular co-ordinates. In fact, if we denote by $\bar{x}$, $\bar{y}$, $\bar{z}$ the special co-ordinates connected with ordinary heliocentric rectangular co-ordinates by the equations

$$\bar{x} = x\left(1 + \frac{k^2\omega^2}{12r^3}\right);\quad \bar{y} = y\left(1 + \frac{k^2\omega^2}{12r^3}\right);\quad \bar{z} = z\left(1 + \frac{k^2\omega^2}{12r^3}\right) \qquad . \qquad (1)$$

where k is the Gaussian constant, ω the interval of integration, and r the radius vector, the fundamental formula of extrapolation (for $\bar{x}$) in the case of perturbed motion is

$$\bar{x}_3 = \bar{x}_2(2 - \sigma_2) - \bar{x}_1 + R_2 + \tfrac{1}{12}\Delta^2 R_2 + f \qquad . \quad . \quad (2)$$

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* B. Noumerov, "Méthode nouvelle de la détermination des orbites et le calcul des éphémérides en tenant compte des perturbations," *Publications de l'Observatoire Astrophysique Central de Russie*, vol. ii., Moscou, 1923.

where σ is determined by the equation

$$\sigma\left(1 - \frac{\sigma}{12}\right)^2 = \frac{k^2\omega^2}{\bar{r}^3} \quad . \quad . \quad . \quad . \quad (3)$$

as a function of $\bar{r}^2 = \bar{x}^2 + \bar{y}^2 + \bar{z}^2$; R is the effect of a perturbing planet with the co-ordinates x', y', z' and the mass m', and is given by

$$R = k^2\omega^2 m'\left(\frac{x' - x}{\rho^3} - \frac{x'}{r'^3}\right) . \quad . \quad . \quad . \quad (4)$$

$\Delta^2 R_2$ is the second difference of R at the second moment, and finally f is the correction for higher derivatives or differences beginning with the 6th order.

$$f = -\tfrac{1}{240}\Delta^6 + \tfrac{31}{60480}\Delta^8 + \ldots . \quad . \quad . \quad (5)$$

If the interval of integration is rightly chosen, it is permissible to neglect the effect of these higher-order terms in formula (2). Subsidiary tables helping to find the interval of integration as well as for determining σ from the equation (3) are given in the above-mentioned work. Computing the co-ordinates with six decimals, it is possible to carry on the integration with intervals of twenty to eighty days for most of the minor planets. As regards comets, especially those with small perihelion distances, it is necessary when they are near perihelion to use a much smaller interval. For economy in time, and in order to reduce the inevitable errors which accumulate during the extrapolation process, it is of great importance in practice to carry on the integration with a long interval. The question of augmenting the interval of integration can be solved in different ways. Firstly, one may consider the question of a more rational choice of special co-ordinates with which the formula (2) would be rigorous to the 8th or 10th order. Secondly, the problem of the systematic accumulation of the influence of higher orders (mainly of the 6th) during the extrapolation process can be solved. It is possible to find the systematic accumulation for any particular distant epoch by rejecting the quantity f, that is to say, by deliberately using a wrong formula. Thirdly, one can consider the question of computing the influence of higher orders from the co-ordinates of three epochs without having recourse to successive approximations. Lastly, the problem of the extrapolation of the perturbations themselves may be solved. It appears, besides, that in view of the smallness of the perturbations the influence of higher orders is very insignificant; that is why it is possible to conduct the extrapolation of perturbations with a longer interval than the extrapolation of the co-ordinates themselves, with the same accuracy. Moreover, the calculations in this case are carried out with a smaller number of figures, which is of great advantage in practice.

Passing over the first three methods, which are sufficiently dealt with in my other papers now ready for publication, I will describe in this paper a method of extrapolating the perturbations of the special co-ordinates, in the main similar to the method of Encke.

Simultaneously with the perturbed motion let us consider the unperturbed, and let us mark all the quantities of the unperturbed

motion with the subscript o. It results from the formula (2) that the second difference of the co-ordinate $\bar{x}$ will be

$$\Delta^2\bar{x} = -\sigma\bar{x} + \mathrm{R} + \tfrac{1}{12}\Delta^2\mathrm{R} + f \quad . \quad . \quad . \quad (6)$$

and similarly for the unperturbed motion

$$\Delta^2\bar{x} = -\sigma_0\bar{x}_0 + f_0 \quad . \quad . \quad . \quad . \quad (6^*)$$

If we denote by $\bar{\xi}$, $\bar{\eta}$, $\bar{\zeta}$ the perturbations of the special co-ordinates

$$\bar{x} - \bar{x}_0 = \bar{\xi}$$
$$\bar{y} - \bar{y}_0 = \bar{\eta}$$
$$\bar{z} - \bar{z}_0 = \bar{\zeta}$$

we find from (6) and (6*)

$$\Delta^2\bar{\xi} = -(\sigma\bar{x} - \sigma_0\bar{x}_0) + \mathrm{R} + \tfrac{1}{12}\Delta^2\mathrm{R} + (f - f_0) \quad . \quad . \quad (7)$$

Denoting

$$\sigma_0 - \sigma = \Delta\sigma \quad . \quad . \quad . \quad . \quad . \quad (8)$$

we have

$$\Delta^2\bar{\xi} = -\sigma\bar{\xi} + \mathrm{R} + \Delta\sigma\bar{x}_0 + \tfrac{1}{12}\Delta^2\mathrm{R} + (f - f_0) \quad . \quad . \quad (9)$$

If we compare the last formula with (6), we can see that the second difference of the perturbations is similar to the second difference of the perturbed co-ordinate $\bar{x}$. The formula (9) differs from the formula (6) by the supplementary term $\Delta\sigma\bar{x}_0$; moreover (and this is of great importance), the formula (9) is influenced only by the *difference* of the higher-order terms in the perturbed and unperturbed motion. Supposing the perturbations to be small and, therefore, that the elements of the osculating ellipse and initial ellipse do not differ much from each other, we see that the difference $(f - f_0)$ will be small and that the formula (9) thus admits a wider interval of integration. This will be of special importance for comets near perihelion, when the perturbations are, generally speaking, very small, but when the extrapolation of the co-ordinates themselves would have to be conducted with very short intervals.

The unperturbed co-ordinates $\bar{x}_0$, $\bar{y}_0$, $\bar{z}_0$ and the quantities $\bar{r}_0$, σ_0 are known from the unperturbed elements of a definite epoch of osculation, $\bar{r}_0$ and σ_0 being here defined by the formulæ

$$\left.\begin{aligned} \bar{r}_0 &= r_0(1 + \tfrac{1}{12}h) \\ \sigma_0 &= \frac{h}{1 + \tfrac{1}{12}h} \end{aligned} \qquad h = \frac{k^2\omega^2}{r_0^{\,2}}\right\} \quad . \quad . \quad . \quad (10)$$

The fundamental formula of extrapolation (9) may be transformed into

$$\bar{\xi}_3 = \bar{\xi}_2(2 - \sigma_2) - \bar{\xi}_1 + \mathrm{R}_2 + \tfrac{1}{12}\Delta^2\mathrm{R}_2 + \Delta\sigma_2\bar{x}_0 + (f - f_0) \quad . \quad (11)$$

in which $\bar{x}_0$ refers to the unperturbed co-ordinate of the second epoch.

It is now necessary to compute the quantity $\Delta\sigma$. From formula (3) applied to the perturbed and unperturbed motions we obtain

$$\left(1-\frac{\Delta\sigma}{\sigma_0}\right)\left(1+\frac{\Delta\sigma}{12-\sigma_0}\right)^2=\frac{1}{\left(1+\frac{\Delta\bar{r}}{\bar{r}_0}\right)} \quad . \quad . \quad . \quad (12)$$

The difference $\Delta\bar{r}=\bar{r}-\bar{r}_0$ is connected with the quantity

$$\pi=\frac{1}{\bar{r}_0^2}\left[\bar{\xi}(\bar{x}_0+\tfrac{1}{2}\bar{\xi})+\bar{\eta}(\bar{y}_0+\tfrac{1}{2}\bar{\eta})+\bar{\zeta}(\bar{z}_0+\tfrac{1}{2}\bar{\zeta})\right] \quad . \quad . \quad (13)$$

by the relation

$$\pi=\frac{\Delta\bar{r}}{\bar{r}_0}+\frac{1}{2}\left(\frac{\Delta\bar{r}}{\bar{r}_0}\right)^2 \quad . \quad . \quad . \quad . \quad (14)$$

Thus we have the perturbations $\bar{\xi}$, $\bar{\eta}$, $\bar{\zeta}$ for any moment and also $\bar{x}_0$, $\bar{y}_0$, $\bar{z}_0$; from (13) we find π, and from (14) $\Delta\bar{r}:\bar{r}_0$, and lastly from (12) we find $\Delta\sigma$ when σ_0 is known.

In consideration of the smallness of the perturbations and the consequent smallness of the quantities π and $\Delta\sigma$, instead of a precise solution by the formulæ (12) and (14), we can make use of a development in series. Indeed, we can write

$$\Delta\sigma=3\sigma_0\pi(1+p) \quad . \quad . \quad . \quad . \quad (15)$$

where p is a small quantity depending on $\sigma_0=4\tau$ and π

$$\left.\begin{aligned}
&p=\alpha+\beta\pi+\gamma\pi^2+\delta\pi^3+\ \ldots\\
&\alpha=\frac{2\tau}{3(1-\tau)}\\
&\beta=-\tfrac{5}{2}\cdot\frac{3-\tau}{3(1-\tau)^3}\left(1-\tfrac{6}{5}\tau+\tfrac{3}{5}\tau^2\right)\\
&\gamma=\tfrac{35}{6}\cdot\frac{3-\tau}{3(1-\tau)^5}\left(1-\tfrac{16}{7}\tau+\tfrac{102}{35}\tau^2-\tfrac{12}{7}\tau^3+\tfrac{3}{7}\tau^4\right)\\
&\delta=-\tfrac{315}{24}\cdot\frac{3-\tau}{3(1-\tau)^7}\left(1-\tfrac{109}{63}\tau+\tfrac{47}{7}\tau^2-\tfrac{484}{63}\tau^3+\tfrac{37}{7}\tau^4-2\tau^5+\tfrac{1}{3}\tau^6\right)
\end{aligned}\right\} \quad . \quad (16)$$

Table I. gives the quantity p to five decimals as a function of the two arguments σ_0 and π. Having computed $\Delta\sigma$ by the formula (15), we find the perturbed value of σ by (8), and thus the fundamental formula of extrapolation (11) allows us to find from the given perturbations of two epochs the perturbation for the third one.

For the computation of the combined effect R from the planets by (4) it is necessary to transform the perturbed special co-ordinates into the ordinary ones, which is done by the formula

$$x=(\bar{x}_0+\bar{\xi})\left(1-\frac{\sigma}{12}\right) \quad . \quad . \quad . \quad . \quad (17)$$

TABLE I.

Values of p *according to Arguments* π *and* σ_0 *in Units of 5th Decimal.*

σ_0 / π	0·00.	0·01	0·02.	0·03.	0·04.	0·05.	0·06.	0·07.	0·08.	0·09.	0·1
−0·010	+2560	+2736	+2914	+3092	+3272	+3452	+3634	+3816	+3999	+4183	+43
−0·009	2298	2474	2650	2828	3006	3186	3366	3547	3729	3913	40
−0·008	2038	2213	2388	2565	2742	2920	3100	3280	3461	3643	38
−0·007	1779	1953	2127	2303	2479	2657	2835	3014	3194	3375	35
−0·006	1521	1694	1868	2042	2218	2394	2571	2749	2928	3108	32
−0·005	1255	1436	1609	1783	1957	2132	2309	2486	2664	2843	30
−0·004	1009	1180	1352	1524	1698	1872	2048	2224	2401	2579	27
−0·003	755	925	1096	1268	1440	1613	1788	1963	2139	2316	24
−0·002	502	671	841	1012	1183	1356	1529	1703	1878	2054	22
−0·001	+ 251	419	587	757	928	1099	1272	1445	1619	1794	19
0·000	000	+ 167	335	504	673	844	1015	1187	1360	1534	17
+0·001	− 249	− 83	+ 84	252	420	590	760	932	1104	1277	14
0·002	− 498	− 332	− 166	+ 1	+ 168	337	506	677	848	1020	11
0·003	− 745	− 580	− 415	− 249	− 82	+ 85	254	423	593	764	9
0·004	− 991	− 827	− 663	− 498	− 332	− 165	+ 2	+ 170	340	510	6
0·005	−1235	−1073	− 910	− 745	− 580	− 415	− 248	− 81	+ 88	257	4
0·006	−1479	−1318	−1155	− 992	− 828	− 663	− 497	− 331	− 163	+ 5	+ 1
0·007	−1722	−1561	−1400	−1237	−1073	− 910	− 745	− 580	− 413	− 246	−
0·008	−1963	−1803	−1643	−1481	−1319	−1156	− 992	− 827	− 662	− 495	− 3
0·009	−2204	−2045	−1885	−1724	−1563	−1400	−1238	−1074	− 909	− 744	− 5
0·010	−2443	−2285	−2126	−1966	−1806	−1644	−1482	−1319	−1156	− 991	− 8

On the other hand, it is also possible to find the perturbations of the ordinary co-ordinates from the perturbations of special ones:

$$\xi = \bar{\xi} + \tfrac{1}{12}(\Delta\sigma\bar{x}_0 - \sigma\bar{\xi}) \quad . \quad . \quad . \quad . \quad (18)$$

which formula can also be employed for computing the initial value of ξ from $\bar{\xi}$ by making use of successive approximations.

The initial values of the perturbations for the first two moments are to be found from a development in powers of the time. If we denote by x_0 the unperturbed co-ordinate and by $x = x_0 + \xi$ the perturbed co-ordinate, we can write the fundamental equation of Encke's method

$$\omega^2\frac{d^2\xi}{dt^2} = -\left(\frac{k^2\omega^2 x}{r^3} - \frac{k^2\omega^2 x_0}{r_0^3}\right) + \mathrm{R} = -(\sigma\bar{x} - \sigma_0\bar{x}_0) + \mathrm{R} \quad . \quad (19)$$

or, after a transformation, similar to (7) and (9), we have

$$\omega^2\frac{d^2\xi}{dt^2} = \mathrm{R} - \sigma\bar{\xi} + \Delta\sigma\bar{x}_0 = \mathrm{F} \quad . \quad . \quad . \quad (19^*)$$

Let us represent the perturbation ξ near the osculation epoch t_0 in the form of a series

$$\xi = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 + \ldots \quad . \quad . \quad (20)$$

It follows from the definition of the epoch of osculation that at $t = t_0$ we must have $\xi = \frac{d\xi}{dt} = 0$, that is to say, from (20) we obtain two equations:

$$a_0 + a_1 t_0 + a_2 t_0^2 + a_3 t_0^3 + \ldots = 0$$
$$a_1 + 2a_2 t_0 + 3a_3 t_0^2 + 4a_4 t_0^3 + \ldots = 0$$

Eliminating the coefficients a_0 and a_1 by means of these last equations, the formula (20) may be written thus:

$$\xi = (t - t_0)^2 \{a_2 + a_3(t + 2t_0) + a_4(t^2 + 2tt_0 + 3t_0^2) \ldots\} \quad . \quad (21)$$

Suppose that we know the right-hand side of the equation (19), namely, F_1, F_2, F_3 for three moments of time t_1, t_2, t_3 near the osculation epoch. In that case we have from (20)

$$\left.\begin{aligned} 2a_2 + 6a_3 t_1 + 12a_4 t_1^2 &= \frac{1}{\omega^2} F_1 \\ 2a_2 + 6a_3 t_2 + 12a_4 t_2^2 &= \frac{1}{\omega^2} F_2 \\ 2a_2 + 6a_3 t_3 + 12a_4 t_3^2 &= \frac{1}{\omega^2} F_3 \end{aligned}\right\} \quad . \quad . \quad . \quad (22)$$

Should we choose the moments at equal intervals ω and suppose for the sake of simplicity $t_2 = 0$, $t_1 = -\omega$ and $t_3 = \omega$, $t_0 = \omega\tau$, by use of the formula (22) the equation (21) can be transformed into the following final form:

$$\xi = \frac{(t - t_0)^2}{\omega^2} \left\{ \frac{F_2}{2} + \frac{F_3 - F_1}{12\omega}(t + 2t_0) + \frac{F_1 + F_3 - 2F_2}{24\omega^2}(t^2 + 2tt_0 + 3t_0^2) \right\} \quad (23)$$

Applying the last formula for three moments t_1, t_2, t_3, we find:

$$\left.\begin{aligned} \xi_1 &= F_1(1, 1) + F_2(1, 2) + F_3(1, 3) \\ \xi_2 &= F_1(2, 1) + F_2(2, 2) + F_3(2, 3) \\ \xi_3 &= F_1(3, 1) + F_2(3, 2) + F_3(3, 3) \end{aligned}\right\} \quad . \quad . \quad . \quad (24)$$

where the coefficients are expressed in terms of

$$\tau = \frac{t_0 - t_2}{\omega} \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

by the formulæ

$$\left.\begin{aligned} (1, 1) &= (3, 3) = \frac{(1 - \tau^2)^2}{8} & (2, 2) &= \frac{\tau^2}{4}(2 - \tau^2) \\ (2, 1) &= \frac{\tau^2}{24}(3\tau^2 - 4\tau) & (2, 3) &= \frac{\tau^2}{24}(3\tau^2 + 4\tau) \\ (3, 1) &= -\frac{(1 - \tau)^2(1 + 2\tau - 3\tau^2)}{24} & (1, 3) &= -\frac{(1 + \tau)^2(1 - 2\tau - 3\tau^2)}{24} \\ (1, 2) &= \frac{(1 + \tau)^2(5 + 2\tau - 3\tau^2)}{12} & (3, 2) &= \frac{(1 - \tau)^2(5 - 2\tau - 3\tau^2)}{12} \end{aligned}\right\} \quad (26)$$

Table II. gives the coefficients (1, 1), (2, 1) . . . (3, 3) as functions of the argument τ.

TABLE II.

Coefficients for computing the Perturbations of Initial Moments in Function of the Argument τ in Units of 5th Decimal.

τ+ve.	(1, 1)=(3, 3).	(1, 2).	(1, 3).	(2, 1).	(2, 2).	(2, 3).	(3, 1).	(3, 2).
0·000	+12500	+41667	−4167	0	+ 0	+ 0	−4167	+41667
0·025	12484	44197	4150	0	31	0	4152	39198
0·050	12438	46787	4100	− 2	125	2	4108	36796
0·075	12360	49433	4012	7	280	8	4040	34461
0·100	12251	52131	3882	16	498	18	3949	32198
0·125	+12112	+54877	−3708	− 30	+ 775	+ 36	−3838	+30007
0·150	11944	57666	3485	50	1112	62	3710	27892
0·175	11746	60496	3211	78	1508	101	3568	25853
0·200	11520	63360	2880	113	1960	153	3413	23893
0·225	11266	66254	2493	158	2467	222	3249	22014
0·250	+10986	+69173	−2034	− 212	+ 3027	+ 309	−3076	+20215
0·275	10681	72112	1511	275	3638	418	2898	18498
0·300	10351	75064	915	349	4298	551	2716	16864
0·325	9999	78025	− 242	433	5002	712	2531	15313
0·350	9625	80987	+ 512	527	5750	902	2346	13846
0·375	+ 9232	+83946	+1354	− 632	+ 6537	+1126	−2162	+12461
0·400	8820	86893	2287	747	7360	1387	1980	11160
0·425	8392	89824	3316	872	8216	1687	1802	9941
0·450	7950	92729	4446	1006	9100	2031	1629	8804
0·475	7496	95603	5683	1150	10008	2422	1462	7748
0·500	+ 7031	+98438	+7031	−1302	+10938	+2864	−1302	+ 6771
τ+ve.	(1, 1)=(3, 3).	(3, 2).	(3, 1).	(2, 3).	(2, 2).	(2, 1).	(1, 3).	(1, 2).

So far as it is possible to neglect the indirect terms in the formula (19) owing to the smallness of perturbations, the function F will be equal to R.

We suppose the perturbations of the special co-ordinates equal to the perturbations of the ordinary ones and compute the value of F by formula (19*). With the help of the corrected value of F we find, using (24), more approximate values of perturbations ξ, η, ζ, and lastly, by means of (18) we determine the perturbations of the special co-ordinates.

This method will now be illustrated by an example of the motion of the planet Erato (62) taken from the work of Oppolzer.*

In Table III. we give the special unperturbed co-ordinates of the planet, $\bar{x}_0$, $\bar{y}_0$, $\bar{z}_0$, to five decimals, computed from the elements.

* Th. R. v. Oppolzer, *Lehrbuch zur Bahnbestimmung der Kometen und Planeten*, B. ii., pp. 105 *et seq.*

Table III.

	N.	$\frac{\bar{x}_0}{\bar{\xi}:2}$.	$\frac{\bar{y}_0}{\bar{\eta}:2}$.	$\frac{\bar{z}_0}{\bar{\zeta}:2}$.	$\frac{\bar{r}_0^2}{\pi.10^7}$.	$\frac{2-\sigma_0}{\Delta\sigma.10^7}$.
1875 ii. 24	1	−2·51804	−2·66282	+0·13867	13·4504	1·9903866
		− 3	+ 1	0	71	2·0
i. 15	2	−2·75005	−2·43244	+0·14075	13·4993	1·9904389
		0	0	0	10	0·3
1874 xii. 6	3	−2·95575	−2·17879	0·14148	13·5036	1·9904434
		0	0	0	12	0·3
x. 27	4	−3·13321	−1·90431	0·14086	13·4632	1·9904003
		− 4	+ 1	0	128	3·7
ix. 17	5	−3·28059	−1·61155	0·13888	13·3787	1·9903091
		− 10	+ 4	0	414	12·1
viii. 8	6	−3·39616	−1·30318	0·13556	13·2506	1·9901680
		− 21	+ 8	0	931	27·5
vi. 29	7	−3·47837	−0·98200	0·13091	13·0805	1·9899753
		− 37	+ 13	0	1743	52·4
. .	.	. .	. .	. .	. .	. .
1872 xi. 6	22	−0·28468	2·76602	−0·05331	7·73475	1·9779090
		− 1249	+ 692	+ 13	59218	3881·3
ix. 27	23	+0·15296	2·72169	−0·06600	7·43535	1·9765561
		− 1352	+ 900	+ 12	61013	4242·9
viii. 18	24	+0·58702	2·61356	−0·07715	7·18124	1·9752955
		− 1421	+ 1144	+ 10	60988	4470·2
vii. 9	25	+1·00656	2·44086	−0·08639	6·97842	1·9742061
		− 1443	+ 1419	+ 5	58844	4506·5

The epoch of osculation is 1874 December 26·0, Berlin M.T.:

$M_0 = 180^\circ\ 40'\ 48''\cdot9$
$\log a = 0\cdot4954793$
$\mu = 640''\cdot89605$
$\phi = 9^\circ\ 59'\ 14''\cdot9$

$\pi = 38^\circ\ 27'\ 17''\cdot9$
$\Omega = 125\ 42\ 39\cdot7$ } 1870·0.
$i = 2\ 12\ 23\cdot9$

The quantities r_0^2, $(2-\sigma_0)$, and $\Delta\sigma$ are also given in Table III.

Table IV. gives the total combined effect of Jupiter and Saturn in units of the 7th decimal ($\omega = 40$), for the first three moments, computed by formula (4). These numbers are taken from Oppolzer's example. The initial values of perturbations for the first three moments near the osculation epoch are determined by the formula (24). In our case $\tau = +0\cdot5$.

In the first approximation the perturbations are found by the formula (22), assuming $F = R$.

TABLE IV.

N.	$\xi \cdot 10^7$.	$\eta \cdot 10^7$.	$\zeta \cdot 10^7$.	$R_x \cdot 10^7$.	$R_y \cdot 10^7$.	$R_z \cdot 10^7$.
1	−658·3	+264·5	+10·1	−528·0	+209·9	−8·1
2	− 75·6	+ 30·4	− 1·2	−584·9	+235·2	−9·0
3	− 78·2	+ 31·5	− 1·2	−646·4	+259·7	−9·7

If we assume the perturbations of the real co-ordinates to be equal to those of the special ones, we find the corrected values of the second derivatives from the perturbations by formula (19).

TABLE V.

N.	$\frac{\bar{x}_0}{\bar{\xi} : 2}$.	$\frac{\bar{y}_0}{\bar{\eta} : 2}$.	$\frac{\bar{z}_0}{\bar{\zeta} : 2}$.	$\frac{\bar{r}_0^2}{\pi \cdot 10^7}$.	$\frac{2-\sigma_0}{\Delta\sigma \cdot 10^8}$.	$R_x \cdot 10^7$.	$F_x \cdot 10^7$.	$R_y \cdot 10^7$.	$F_y \cdot 10^7$.	$R_z \cdot 10^7$.	$F_z \cdot 10^7$.
1	−2 51804	−2·66282	+0·13867	13·4504	1·9903866	−528·0	−526·7	+209·9	202·1	−8·1	−7
	− 3	+ 1	0	71	20	− 5·0		− 5·3		+0·3	
						+ 6·3		− 2·5		+0·1	
2	−2·75005	−2·43244	+0·14075	13·4993	1·9904389	−584·9	−585·0	+235·2	234·2	−9·0	−9
	0	0	0	10	03	− 0·8		− 0·7		0·0	
						+ 0·7		− 0·3		0·0	
3	−2·95575	−2·17879	+0·14148	13·5036	1·9904434	−646·1	−646·3	+259·7	258·8	−9·7	−9
	0	0	0	12	03	− 0·9		− 0·6		0·0	
						+ 0·7		− 0·3		0·0	

With the improved values of F we recompute the perturbations of ordinary co-ordinates ξ, η, ζ by the formula (24), then from (18), with the values given in Table V., we find the perturbations of the special co-ordinates $\bar{\xi}$, $\bar{\eta}$, $\bar{\zeta}$.

TABLE VI.

N.	$\xi \cdot 10^7$.	$\bar{\xi} \cdot 10^7$.	$\eta \cdot 10^7$.	$\bar{\eta} \cdot 10^7$.	$\zeta \cdot 10^7$.	$\bar{\zeta} \cdot 10^7$.
1	−658·3	−658·4	263·0	263·6	−10·1	−10·1
2	− 75·6	− 75·6	30·4	30·5	− 1·2	− 1·2
3	− 78·2	− 78·2	31·4	31·5	− 1·2	− 1·2

Having thus obtained the perturbations for the first two moments, we proceed with the extrapolation by formula (11), in the first place finding the perturbations for the third moment as a numerical check. The computation of the perturbations is given in Tables VII. and III. The combined effects were borrowed from Oppolzer's example, and are corrected for second differences.

The perturbations are computed successively by (13), (15), and (16), and the multiplier p is taken from Table I. as a function of the two arguments π and σ_0.

In the same Table VII. we give the final values of the perturbations ξ, η, ζ of the ordinary rectangular ecliptic co-ordinates of the planet

Erato (62), computed by formula (18). The differences $\Delta\xi$, $\Delta\eta$, $\Delta\zeta$ from Oppolzer's results are given in the table.

TABLE VII.

N.	$\frac{R_x}{\Delta\sigma\,.\,\bar{x}_0}$.	$\frac{\bar{\xi}\,.\,10^7}{\xi-\bar{\xi}}$.	$\frac{\xi\,.\,10^7}{\Delta\xi}$.	$\frac{R_y}{\Delta\sigma\bar{y}_0}$.	$\frac{\bar{\eta}\,.\,10^7}{\eta-\bar{\eta}}$.	$\frac{\eta\,.\,10^7}{\Delta\eta}$.	$\frac{R_z}{\Delta\sigma\,.\,\bar{z}_0}$.	$\frac{\bar{\zeta}\,.\,10^7}{\zeta-\bar{\zeta}}$.	$\frac{\zeta\,.\,10^7}{\Delta\zeta}$.	$\frac{2-\sigma}{\sigma:12}$.
1	− 528·0	− 658·4	− 658·3	+209·9	263·6	263·3	− 8·1	− 10·1	− 10·0	1·9903868
	− 5·0	+ 0·1	0	− 5·3	− 0·6	0	+ 0·3	0·1	0	0·00080110
2	− 585·3	− 75·6	− 75·6	235·1	30·5	30·4	− 9·0	− 1·2	− 1·2	1·9904389
	− 0·8	+ 0·0	0	− 0·7	− 0·1	0	0	0	0	79676
3	− 646·4	− 78	− 78	259·7	31	31	− 9·7	− 1	− 1	1·9904434
	− 0·9	0	0	− 0·7	0	+1	0	0	0	79638
4	− 711·2	− 727	− 727	281·2	290	289	− 10·0	− 10	− 10	1·9904007
	− 11·6	0	0	− 7·0	− 1	+2	0·5	0	− 1	79994
5	− 778·3	− 2092	− 2093	298·2	820	818	− 9·7	− 29	− 29	1·9903103
	− 39·7	− 1	0	− 19·5	− 2	+2	+ 1·7	0	− 1	80747
6	− 844·8	− 4255	− 4259	307·6	1621	1617	− 8·7	− 56	− 56	1·9901708
	− 93·4	− 4	0	− 35·8	− 4	+3	+ 3·7	0	− 1	81910
7	− 906·9	− 7314	− 7323	306·8	2678	2671	− 6·9	− 87	− 86	1·9899805
	− 182·2	− 9	0	− 51·5	− 7	+3	+ 6·9	+ 1	− 1	83496
	.	.	.	.	.	.	.	.	.	.
22	− 194·7	−219874	−249454	− 36·6	138459	139103	8·7	2636	2614	1·9782971
	−1104·9	+ 360	− 10	10735·8	+644	− 1	−206·9	− 22	+ 2	180857
23	− 156·4	-270323	−269750	− 39·0	179996	180613	7·3	2452	2424	1·9769804
	+ 649·0	+ 573	− 9	11547·8	+617	0	−280·1	− 28	+ 2	191830
24	− 124·7	−284117	−283324	− 42·0	228898	229409	6·0	1939	1903	1·9757426
	+2624·0	+ 793	− 7	11683·9	511	0	−344·9	− 36	+ 2	202146
25	− 98·9	−288520	−287533	− 45·5	283889	284206	4·9	1040	1005	1·9746568
	+4536·6	+ 987	− 4	11000·1	+317	+1	−389·3	− 35	+ 2	21193

The divergence at the end of twenty-five steps of extrapolation does not exceed ten units of the 7th decimal, which are due to cumulative errors during the process of numerical integration.

The method here described can be applied not only to the computation of the perturbations, in comparison with elliptic motion, but generally for finding the differences between two perturbed motions. For instance, one can in this way take into account separately the perturbations caused by Saturn, or the systematic effect of higher-order terms neglected in the extrapolation of perturbed co-ordinates.

Petrograd,
The Astronomical Observatory
of the Petrograd University:
1924 *January.*

Note on the Numerical Integration of $\frac{d^2x}{dt^2}=f(x, t)$.

By J. Jackson, M.A., D.Sc.

After the discovery of the eighth satellite of Jupiter, Cowell introduced numerical integration into dynamical astronomy as a method for taking account of all the forces, including the action of the central body. This method he applied later to the investigation of the motion of Halley's Comet between 1759 and 1910. This work of Cowell's has led other workers to deal with the subject of numerical integration, as it affords a ready solution to problems otherwise intractable. Cowell, instead of using the formulæ for numerical integration which had been left in a practically perfect state by Gauss,* reinvestigated the problem and introduced formulæ for changing the second differential coefficients into second differences, so that the integral was obtained at once by a simple double summation. After the work had been done, Cowell found out the disadvantages of the method he introduced, and incorporated the long-established formula in the first of two suggestions for future computations, which are given at the end of the essay by himself and Crommelin on the motion of Halley's Comet, published as an appendix to the Greenwich Observations for 1909. As these suggestions do not appear to be well known they are repeated here.

(1) In place of the fundamental formula

$$x_2=\mathrm{X}+\tfrac{1}{12}\mathrm{X}_2-\tfrac{1}{240}\mathrm{X}_4+\tfrac{1}{1951}\mathrm{X}_6\ \ldots$$

it would have been far better to use

$$x=\mathrm{X}_{-2}+\tfrac{1}{12}\mathrm{X}-\tfrac{1}{240}\mathrm{X}_2+\tfrac{1}{1951}\mathrm{X}_4.\ \ldots$$

(2) Short time intervals ought to be employed, as errors in that case are far easier to detect. The intervals in the *Ast. Ges.* essay were far too long, and those in the revised calculations of this appendix not too short. The intervals should be 1 day at perihelion, and may be doubled at 40, 80, 160 days, and so on from perihelion. The interval should not be changed during a close approach to a planet.

Of these suggestions (1) is a return to a long-established formula and (2) is a point of practical importance, the truth of which is borne home with greater and greater force the more one computes. A numerical mistake made at any point may vitiate all subsequent work. Professor Noumerov has recently introduced "special co-ordinates" in

* The formulæ for numerical integration go right back to the beginnings of the calculus. Formulæ were developed by Newton, Gregory, and others in the seventeenth and eighteenth centuries. The formulæ advocated here were known to Gauss early in the nineteenth century. The subject was carefully discussed by Encke in the Berlin *Jahrbuch* for 1837, and is given in great detail by Oppolzer at the beginning of the second volume of his *Lehrbuch zur Bahnbestimmung der Kometen und Planeten.*

an attempt to get over the difficulties of extrapolation required in estimating the values of X_2, X_4, . . . in the formulæ given above. It will be seen that these special co-ordinates make the first and largest (though not necessarily the most important) step backwards indicated by Cowell in (1) above. Noumerov has then attempted to introduce the special co-ordinates into the differential equation, with the result that his formulæ appear rather complicated. It does not appear that the long-established formulæ can be improved.

Notation.—The fundamental formulæ, although long known, are so rarely given in English textbooks that they may be given here, together with a convenient notation. The values actually computed from the differential equation, at equal intervals of the time, are denoted by $f(1), f(2), f(3)$, etc. The values are given below in a vertical column, and the differences formed according to the general formula $f_{n+1}(t) = f_n(t+\frac{1}{2}) - f_n(t-\frac{1}{2})$ are given to the right. The sums are given to the left, and are computed from ${}_{n+1}f(t) = {}_{n+1}f(t-1) + {}_nf(t-\frac{1}{2})$. The quantities in brackets $[{}_2f(1)]$ and $[{}_2f(2)]$ have to be chosen to suit the initial values of x and its velocity. In the actual mechanical integration the numerical values of the functions are tabulated according to the schemes:

$[{}_2f(1)]$		$f(1)$				
	${}_1f(\frac{3}{2})$		$f_1(\frac{3}{2})$			
$[{}_2f(2)]$		$f(2)$		$f_2(2)$		
	${}_1f(\frac{5}{2})$		$f_1(\frac{5}{2})$		$f_3(\frac{5}{2})$	
${}_2f(3)$		$f(3)$		$f_2(3)$		$f_4(3)$.
	${}_1f(\frac{7}{2})$		$f_1(\frac{7}{2})$		$f_3(\frac{7}{2})$	
${}_2f(4)$		$f(4)$		$f_2(4)$		
	${}_1f(\frac{9}{2})$		$f_1(\frac{9}{2})$			
${}_2f(5)$		$f(5)$				

If we choose $[{}_2f(1)]$ and $[{}_2f(2)]$, so that

$$x(1) = [{}_2f(1)] + \tfrac{1}{12}f(1) - \tfrac{1}{240}f_2(1) + \tfrac{31}{60480}f_4(1) \ . \ . \ .$$
$$x(2) = [{}_2f(2)] + \tfrac{1}{12}f(2) - \tfrac{1}{240}f_2(2) + \tfrac{31}{60480}f_4(2) \ . \ . \ .,$$

then we have for all integral values of n

$$x(n) = {}_2f(n) + \tfrac{1}{12}f(n) - \tfrac{1}{240}f_2(n) + \tfrac{31}{60480}f_4(n) \quad . \quad . \quad (1)$$

Cowell and Crommelin in their work, instead of applying the correcting terms to ${}_2f(n)$, applied them to $f(n)$, so that the double summation gave the co-ordinate directly by double summation, *i.e.* they computed

$$X(n) = f(n) + \tfrac{1}{12}f_2(n) - \tfrac{1}{240}f_4(n) + \tfrac{31}{60480}f_4(n) \quad . \quad . \quad (2)$$

and then obtained x from a double summation, thus

$$\begin{array}{ccc} x(1) & & X(1) \\ & {}_1X(\tfrac{3}{2}) & \\ x(2) & & X(2) \\ & {}_1X(\tfrac{5}{2}) & \\ x(3) & & X(3). \end{array}$$

It is clear that this procedure is exactly similar to that advocated in formula (1), except that the "correcting terms" in (2) are obtained by differencing twice more than for (1) and have to be finally summed twice. The serious disadvantages in the use of (2) as compared with (1) lies in the influence of neglected higher differences, for any error made in deducing $X(n)$ from $f(n)$ enters into ${}_1X(n+\frac{1}{2})$ and all subsequent values of this function and produce steadily increasing errors in x. On the other hand, neglected terms in the deduction of $x(n)$ from ${}_2f(n)$ by (1) are not passed on to subsequent values of x. In fact, $x(n)$ as required for computing $f(x, t)$ is not required very accurately, as the time interval should be chosen so that $f(x, t)$, which is roughly proportional to x, does not usually exceed in numerical value $x/100$.

Professor Noumerov has used a "special co-ordinate," $\bar{x} = x - \frac{1}{12}f(x, t)$, so that $\bar{x}$ differs from ${}_2f(n)$ by a term beginning with $\frac{1}{240}f_2(n)$. In the paper just presented to the society (p. 592) he writes about the possible extension of the use of "special co-ordinates," so as to make the differential coefficient approximate more closely still to the second difference. When he has completed the process he will find that $\bar{x}$ is the ${}_2f(n)$ of formula (1).

The trouble in the use of all these formulæ is the extrapolation necessary for the determination of the higher differences required for converting the differential coefficients into differences or the sums into integrals. The great advantage of formula (1) over the others is that this extrapolation only affects the deduction of $x(n)$ from ${}_2f(n)$, so that in all cases ${}_2f(n)$ is accurate, while $x(n)$ does not need to be very accurately known for the computing of $f(n)$, which leads accurately to ${}_2f(n+1)$ at once. It may seem that the need to apply so large a correction as $\frac{1}{12}f(n)$ to ${}_2f(n)$ in order to obtain $x(n)$ by formula (1) would lead to considerable uncertainty, for $f(n)$ may contain six or even seven significant figures. But when the arithmetic is carried out with a suitable time interval this can be done with all the required accuracy. Suppose the differences fall off in such a way that f_6 can be extrapolated without serious error. Then, when $f(n-1)$ is known so also are the values of f_6 to $f_6(n-4)$. When once we have extrapolated $f_6(n-3)$ we can compute with equal accuracy $f_5(n-\frac{5}{2})$, $f_4(n-2)$, $f_3(n-\frac{3}{2})$, $f_2(n-1)$, $f_1(n-\frac{1}{2})$, $f(n)$. An error of 12 units in the last decimal place of the extrapolated value of $f_6(n-3)$ will only produce 1 unit in the last decimal place of $x(n)$. Another extrapolation of f_1 to $f_6(n-2)$ will enable $f_2(n)$ to be computed, and only ${}_2\frac{1}{4}{}_0$th part of this has to be applied in formula (1). A little consideration will show that the unknown terms of (1) can be extrapolated with all the accuracy required.

As we have seen, Professor Noumerov has made the first step in the transition form (2) to (1) by introducing $x(n) - \frac{1}{12}f(n)$ as a "special co-ordinate" $\bar{x}$. By making use of the fact that except for perturbations $f(x, t) = -k^2x/r^3$, and similar expressions for the other three co-ordinates, he deduces $\frac{d^2x}{dt^2} = \sigma\bar{x}$ where

$$\sigma(1 - \tfrac{1}{12}\sigma)^2 = k^2/\bar{r}^3,$$

and

$$\bar{r}^2 = \bar{x}^2 + \bar{y}^2 + \bar{z}^2.$$

$\frac{d^2x}{dt^2}$ is thus computed by means of $\bar{x}$, $\bar{y}$, $\bar{z}$ without transforming to x, y, z.

Professor Noumerov has taken considerable trouble to apply the formulæ which use $\bar{x}$, $\bar{y}$, $\bar{z}$, and the success he has met with has been due to the use of the above cubic equation in σ. His process could not be easily extended so as to use ${}_2f(n)$ as a "special co-ordinate" in place of x. As it is, his formulæ are rather involved compared with the original one, and the process for deriving $\bar{x}_n$ from earlier values much more complicated. Experience has shown that (1) can be applied with ease, and it appears that attempts to improve it will complicate its application so much as to increase the labour of integration.

As an example, take the values of $\frac{d^2x}{dt^2}$ given by Cowell and Crommelin at the beginning of their discussion of Halley's Comet. The time interval is 32 days, and the first date is 230 days after the perihelion of 1759. The interval is rather long, as is shown by the fact that f_6 is changing rather rapidly, and in fact Cowell advocated that the interval should be reduced. From nine values of f, three values of f_6 can be obtained. These three values indicate that f_6 is being approximately halved at each step, and in the table we have extrapolated other three steps, assuming that f_6 is actually halved each time. We have then computed three extra values for all the other differences, although they have not been bracketed.

f.	f_1.	f_2.	f_3.	f_4.	f_5.	f_6.
+2290 736						
1881 663	−409 073					
1581 200	300 463	+108 610				
1353 121	228 079	72 384	−36 226			
1175 289	177 832	50 247	22 137	+14 089		
1033 533	141 756	36 076	14 171	7 966	−6 123	
918 405	115 128	26 628	9 448	4 723	3 243	+2 880
823 415	94 990	20 138	6 490	2 958	1 765	1 478
743 960	79 455	15 535	4 603	1 887	1 071	694
(676 600)	67 360	12 095	3 440	1 163	724	(347)
(618 508)	58 092	9 268	2 827	613	550	(174)
(567 007)	51 501	6 591	2 677	150	463	(87)

When we form the sums we get the following values, the first column being a repetition from above:

f.	$_1f$.	$_2f$.
+2290 736		
1881 663	−29 522 122	
1581 200	27 640 459	3·66 917 633
1353 121	26 059 259	3·94 558 092
1175 289	24 706 138	4·20 617 351
1033 533	23 530 849	4·45 323 489
918 405	22 497 316	4·68 854 338
823 415	21 578 911	4·91 351 654
743 960	20 755 496	5·12 930 565
(676 600)	20 011 536	5·33 686 061
(618 508)	19 334 936	5·53 697 597
(567 007)	18 716 428	5·73 032 533
	18 149 421	5·91 748 961
		6·09 898 382

The last four values of $_2f$ are extrapolations beyond the dates for which x has been used. The value of x computed from $_2f$ for three dates beyond those used are:

$_2f$	−5·53 697 597	−5·73 032 533	−5·917 48 961
$+\frac{1}{12}f$	+ 56 383·3	+ 51 542·3	+ 47 250·6
$-\frac{1}{240}f_2$	− 38·6	− 27·5	− (16·4)
$+\frac{1}{1061}f_4$	+ ·1	...	...
	−5·53 641 252	−5·72 981 018	−5·917 01 727

All these values are extrapolated except the first value of $_2f$.

The final values obtained by Cowell and Crommelin were

−5·53 641 244	−5·72 980 858	−5·917 00 633

It will be seen that the first two extrapolated values are sufficiently accurate for the computation of the next two values of $\frac{d^2x}{dt^2}$, although in reality we only need to extrapolate one step at a time. The values of f_6 obtained by actual differentiation of the tabulated values are 455, 203, 169, instead of 347, 174, 87 assumed above. Of course slight errors in the computed values of $\frac{d^2x}{dt^2}$ affect f_6 considerably.

Some Eye-pieces made by Sir William Herschel.

By W. H. Steavenson. (Plate 17.)

The claims made by Sir William Herschel to have used powers up to and over 6000 on one of his 7-foot telescopes have always aroused the interest and curiosity of practical astronomers. From the title * and substance of a paper written in 1782, and published in *Phil. Trans.*, 72, pp. 173–178, it is evident that from the very first there had been considerable hesitation on the part of scientific men in accepting his statements on this point. In this paper Herschel gives the results of a re-measurement of his powers by a method different from that formerly used. The powers, as thus re-measured, come out substantially the same, and in fact the values are all actually higher except for the power previously referred to as "6450," which was now found to be 5787. He goes on to point out that the use of such powers was not his regular habit, but was rather in the nature of an experiment, and that "farther inferences ought not as yet to be drawn." But, in spite of the very clear and circumstantial statements contained in this paper, some incredulity with regard to the matter has always persisted, and in modern text-books it is common to find doubts expressed as to the possibility of the construction and use of such lenses, or of the measurement of their powers.

In the month of May 1924, with the kind permission of the Rev. Sir John Herschel, I began a systematic examination of such of the instruments of Sir William Herschel as are still preserved at Slough. Among those already recognised as the handiwork of Sir William were many eye-pieces in brass and wooden mounts, but none of the extremely high powers already mentioned. However, while examining the contents of a cabinet, supposed to contain only some chemical specimens and apparatus used by Sir J. F. W. Herschel, I came across nine small wooden cylinders, about 0·8 by 0·9 inch. A closer examination showed these to be eye-pieces fitted with lenses of exceedingly short focal length, and I think there can be no doubt but that these are some of the famous high powers made and used by Sir William.

All are constructed on the same plan. The main body of each is of cocus wood, turned no doubt by Herschel on the lathe which he used for all such work, and which is still preserved at Slough in excellent condition. This same wood was used for many of the larger eye-pieces, and it seems probable that its use as a suitable material was first suggested to Herschel by his familiarity with the oboe, for the making of which instrument it was, and still is, largely employed. At the lower end of each mount is cut a thread, of the same standard pitch and diameter as that used for many other of his eye-pieces, both brass and wood. On the upper end of each cylinder there screws down a wooden rim, which serves to hold in position two circular plates of very thin brass. These plates, which are exactly alike, have all

* "A Paper to obviate some Doubts concerning the great Magnifying Powers used." By Mr. Herschel, F.R.S.

evidently been stamped out by the same die. In shape they are very much like the central portions of a pair of cymbals; that is, they have domed centres surrounded by a flat rim. In the middle of each dome is a smaller elevation, in the centre of which a hole of appropriate size is bored. The plates were so arranged by their maker as to bring the holes exactly opposite to one another, the lower one being in general the smaller of the two. A V-shaped cut was then made through both plates at the rim, so that they could always be readily re-centred if disarranged.

Held between the two holes, and of somewhat greater diameter than the larger, is a single bi-convex lens. After the lapse of well over a century since the last time of their being used, there was naturally a considerable accumulation of dirt on the surfaces of these lenses. They were therefore carefully and separately dismounted, and cleaned by immersion in a single drop of nitric acid. After remounting, each was examined with a microscope and its focal length determined with a focometer, the probable error in each case being about ·0005 inch. The following table gives the focal lengths of the first eight eye-pieces as thus measured, together with the powers which they would give on a telescope of 85·2 inches focal length, Herschel's experiments having been made with such an instrument. Seven of the eye-pieces had already been marked in pencil with their supposed powers, either by Sir William or by Sir J. F. W. Herschel, but there is nothing to show for what focal length of speculum they are calculated. Probably a telescope of about 7-foot focus is intended, and the errors are such as might be expected from the comparatively rough methods used so long ago in measuring the focal lengths.

Eye-piece.	Focal Length.	Power on 85·2 in. Focal Length.	Power as Marked.
	inches		
A	0·064	1331	"1504"
B	0·046	1852	"2100"
C	0·038	2242	"2589"
D	0·024	3550	"4140"
E	0·023	3704	[Not marked]
F	0·0225	3787	"4294"
G	0·019	4484	"4823"
H	0·0175	4869	"5489"

It is worth noting that each of the above is a perfectly formed lens with bevelled edge, with the exception of D, which is a simple sphere, made probably by Della Torre's method of fusion. The surfaces of some of them are slightly devitrified through age, but the images formed by them of distant objects are quite sharp as seen through a microscope, with a power of 100 diameters.

The ninth lens, J, is of particular interest, as being the smallest and most powerful yet found among Herschel's eye-pieces. Like the

others, it is a well-formed bi-convex lens, but its total diameter is only 0·02248 in., or about $\frac{1}{45}$ in., while its greatest thickness is 0·01091 in., or $\frac{1}{92}$ in. Its focal length, measured carefully several times, is 0·0111 in., or $\frac{1}{90}$ in. The upper and lower apertures in the plates between which it is mounted are, respectively, $\frac{1}{57}$ in. and $\frac{1}{92}$ in. in diameter. The magnifying power, as given on a focal length of 85·2 in., would be 7676. The surfaces are not truly spherical, being somewhat hyperboloidal in figure. The axes of the two are, unfortunately, neither coincident nor quite parallel, with the result that the lens is somewhat astigmatic. However, it forms fair images, as seen under the microscope. There is no mark on it to indicate its reputed power.

Out of curiosity I used it as an eye-piece on my 6-inch Wray refractor, whose focal length is 112·5 in., the resultant power being about 10,000. The field of view, defined roughly by the lower pin-hole, was some twenty seconds of arc in diameter. One ansa of Saturn's ring and about half the ball could be seen at one time. The image was naturally dim and diffuse, but the general contour of the inner and outer edges of the ring could be made out quite well, and it was just possible to see across which apparent hemisphere of the ball it passed. The Cassini division could not be seen. Vega, observed with the same eye-piece, was seen as a large and brilliant disc, with diffuse edges. The first dark space and first diffraction ring were clearly visible, though the images that night were not particularly good, even with low powers.

There is no mention in any of Sir William Herschel's published works of a power so high as this. Perhaps he may never have used it after its first trial, as it is less perfect than some of those of lower power. On the other hand, it is quite possible that it is actually identical with the power referred to by him as "6450," for the astigmatism of the lens has the effect of enlarging, in certain position angles, the images formed by it. This enlargement is in the ratio of 6 : 5 as compared with the scale given by other meridians, and might thus give rise to an erroneous value of ·0133 in. for the focal length, corresponding to a power of 6406. In any case it is difficult to see how such a lens could have been used on a telescope mounted on an altazimuth stand, with slow motions worked by hand. Even on a focus of 7 feet the field of view would have been only about 26 seconds of arc in diameter, so that an equatorial star would take less than two seconds of time to pass right across it. To find, hold, and examine an object under such conditions must have been well-nigh impossible.

But, quite apart from any question of its practical utility in observation, this little eye-piece is at least an optical curiosity of the greatest interest. It must be one of the smallest true lenses ever figured, and the production of a similar one to-day would reflect the highest credit upon any optician. Made more than a hundred and forty years ago, it must be regarded as a triumph of manipulative skill.

EXPLANATION OF PLATE 17.

1. The eye-piece of highest power, showing the wooden mount, outer brass plate, pin-hole aperture, and lens. Enlarged about two diameters.
2. Lens from the above, dismounted, cleaned, and viewed approximately at right angles to its axis. The two small projections from the upper surface are minute specks of dust, less than $\frac{1}{1000}$ in. in diameter. The whole photographed through a microscope, the enlargement being 58 diameters.
3. Image formed by the (mounted) lens when used as a photographic objective. The unequal definition of lines in different position-angles is a result of astigmatism, the blurring at the lower end of the telescope being further increased by a speck of dust. The boundary corresponds to the internal aspect of the far end of the mount. The astigmatism of the lens has converted it from a circle into an ellipse. The aerial image formed by the lens appears as photographed through a microscope, the enlargement being 210 diameters.

The Rotation Period of the Sun as derived from Magnetic Storms.

By E. Walter Maunder and A. S. D. Maunder.

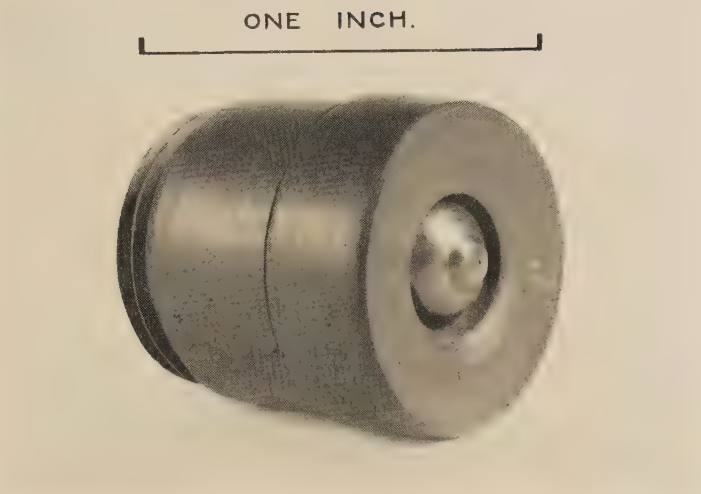
On March $21^d\ 14^h$ (G.C.T.) the centre of the great group of 1920 crossed the central meridian of the sun; on the 22nd, at a few minutes past 8 a.m., a magnetic storm began sharply; the following day, March 23, at Calama, in Chile, the "solar constant" showed a marked depression.* This group (9143 in the Greenwich series) was a revival of a spot (9127) which had crossed the central meridian on February $23^d\ 11^h$, and had been followed by a considerable magnetic storm which began sharply at about $8^h\ 30^m$ a.m. on February 24; this again was accompanied by a very slight depression of the solar constant on the same day. Dr. Abbot finds that it is "almost unfailing for a minimum of radiation" to fall about "a day after the central transit of the most strongly spotted area."† The intervals between the returns of the spot groups, of the magnetic storms and of the depression of the solar constant, are $27^d{\cdot}1$, $27^d{\cdot}0$, and 28^d respectively.

The intervals given by the spots and by the magnetic storms are in close accord, but they are in closer agreement with the rotation period given by spots in south latitude 15° to 17° than with that in 5° ($26^d{\cdot}83$), which is the mean latitude of 9143.‡ Though it is true that when we are dealing with the *mean* motions in any zone the rotation period generally lengthens with the distance of the spot from the equator, it by no means follows that the individual group giving the longest period is situated in the highest latitude. The bearing of this fact upon the relation between any solar group and a magnetic storm is obvious.

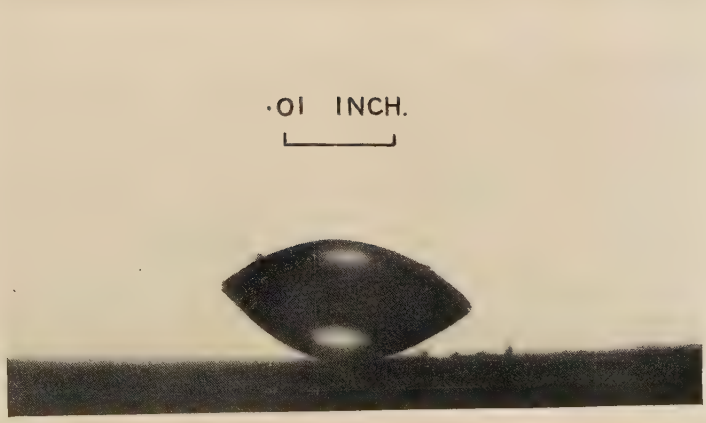
Another such example of the scattering ‡ of groups situated in any particular latitude zone through a wide range of rotation periods is given by an observation made of the great prominence of 1919 May, by Mr. Evershed.§ The preceding end of one portion of the prominence,

* *Annals Smithsonian Institution*, **4**, 186.
† *Nat. Acad. Sci. Proc.*, **9**, 357.
‡ *M.N., R.A.S.*, **65**, 8, pp. 823 and 817.
§ *Ibid.*, **80**, 1, p. 10.

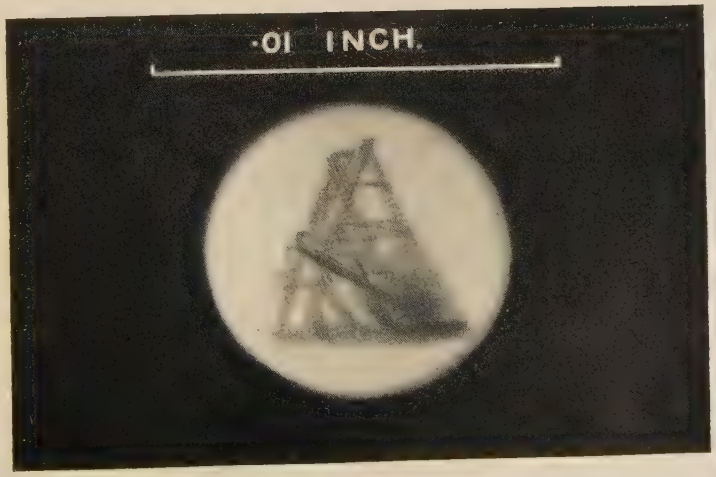
1



2



3



seen as a dark absorption marking, in latitude -15° (being about 14,000 miles above the level of the photosphere) gave a rotation with the same angular speed as a spot in north latitude 7°, the two maintaining throughout the same relative positions in longitude, and completing an entire synodic rotation in 27·02 days; the mean synodic rotation of recurrent spots in north latitude 7° being $26^d{\cdot}76$. From this Mr. Evershed drew the conclusion: "The prominence-producing regions rotate with the angular speed of the spots and the reversing layer."

The close accordance of the various forms of solar activity in mean daily motion becomes evident when we take this for the most active zone ($12\frac{1}{2}^\circ$) of spots, faculæ, and flocculi. The last two lines of the table give the *mean* rotation periods for all spots and for recurrent spots irrespective of latitude.

TABLE I.

Phenomenon.	Authority.	Daily Sidereal Motion.	Sidereal Rotation Period.	Synodic Rotation Period.
		°	d	d
Spots (6 to 14 days)	Maunder *	14·45	24·92	26·74
Faculæ	Newton †	14·42	24·96	26·79
Flocculi	Fox ‡	14·38	25·04	26·89
Recurrent spots	Maunder *	14·34	25·11	26·95
Faculæ	Chevalier §	14·33	25·13	26·98
All spots	Maunder *	14·40	25·00	26·83
Recurrent spots	Maunder *	14·29	25·20	27·07

The extreme range in the sidereal period is but the fifth of a day, yet these phenomena are at different levels on the sun. Spots are on or below the photosphere, faculæ and flocculi are distinctly elevations of it. They often overflow spots, but the reverse action never takes place, for a spot does not invade or hide a facula or flocculus.

These various phenomena on the sun are distinct from each other, but the rotation periods they give are not distinct. Therefore it is important to analyse the rotation periods as given by phenomena, which are measured by effects on the earth, not by movements on the sun. It has been demonstrated || that magnetic disturbances tend to be repeated after intervals corresponding closely to the mean rotation period of the sun as derived from sunspots. These events take place on the earth though they have their origin on the sun; and this solar origin being assumed in the present paper, it is proposed to determine the value of the solar rotation period most favoured by magnetic storms.

The following table is an adaptation of one given on p. 817 of *M.N., R.A.S.*, **65**, 8. The number of spot groups yielding the synodic periods (at intervals of a fifth of a day) are given for the northern

* *M.N., R.A.S.*, **65**, 8, pp. 816, 823, and 824.
† *Ibid.*, **84**, 6, p. 440.
‡ *Publications of the Yerkes Observatory*, **3**, 159.
§ *Astrophysical Journal*, **32**, No. 5, p. 398.
|| *M.N., R.A.S.*, **65**, 1, pp. 2–34; 6, pp. 538–559; 7, pp. 666–681.

and southern solar hemispheres respectively, the number of magnetic storms yielding the same synodic period being placed between them.

TABLE II.

Synodic Rotation Period. d	Spot Numbers. North. 1879-1901.	Magnetic Storms. Pairs.	Spot Numbers. South. 1879-1901.	Total of Spots. N. and S.
25·0	12	...	16	28
·2	17	...	16	33
·4	15	1	22	31
·6	28	0	24	52
·8	42	2	31	73
26·0	42	8	46	88
·2	53	9	70	123
·4	66	7	67	133
·6	66	8	92	158
·8	85	19	85	170
27·0	98	30	121	219
·2	93	32	121	214
·4	73	10	86	159
·6	44	10	58	102
·8	32	20	46	78
28·0	18	13	22	40
·2	14	7	21	35
·4	11	6	19	30
·6	7	6	5	12
·8	3	2	11	14
29·0	5	...	2	7
·2	1	...	4	5
·4	1	...	2	3

It is obvious on inspection that the magnetic storms cluster at the synodic periods $26^d{\cdot}8 - 27^d{\cdot}2$, a sharp maximum occurring at $27^d{\cdot}0 - 27^d{\cdot}2$. This is also the synodic period most favoured by both the northern and southern spots. The mean synodic period given by spot groups of 6 to 14 days of life is $26^d{\cdot}83$, by recurrent groups it is $27^d{\cdot}07$. Hence we conclude that the mean rotation period of the sun as derived from magnetic storms here on the earth is—within the limits of errors of measurement—the same as that given by spots on the sun, and therefore also the same as that given by faculæ, flocculi, prominences, or reversing layer.

In 1904, one of the authors of this paper wrote* concerning the manner of this action from the sun on the earth: "The influence (which produces magnetic storms) proceeding from the sun, whatever

* *M.N., R.A.S.*, 65, 1, pp. 30–31.

its character, does not act equally in all directions. It does not radiate like light and heat, but its action is confined to a definite and very restricted direction. This appears from a consideration of the characteristic 'sharp' movement with which so many magnetic disturbances, and all the more violent of them, begin. It would be possible to account for this sudden and instantaneous effect, instantaneous over all the earth, as the impact of a wave of energy radiating in all directions from the sun as centre, if such storms bore no relation to each other. It is not possible so to account for such an effect when it is followed by others exactly at the interval of one or more synodic rotation-periods of the sun. Such a relation can only be explained by supposing that the earth has encountered, time after time, a definite stream, a stream which, continually supplied from one and the same area of the sun's surface, appears to us at our distance to be rotating with the same speed as the area from which it rises."

Streams behaving in this manner have been photographed as coronal rays. The base of the ray is broad, covering a considerable area of the photosphere. They are fed by electrified particles,* both positively and negatively charged, which incline towards an apex, growing always more pointed, until it is forced out into a long rod-like ray. These rays, when photographed during an eclipse, remain straight, narrow, and defined up to the limit of their visibility on the plate.

These streams of charged particles show by their action at the earth that they are moving at this distance from the sun—one astronomical unit—with the same angular velocity as the area of disturbance on the sun which expels them. This area is not the entire surface of the sun, but a restricted area of the photosphere. It does not always move with the same angular velocity, and the action at the earth's distance appears to respond to the change in velocity.

The same sort of action by restricted streams has been followed to a distance from the sun of nearly ten astronomical units. In 1920, between January and May, Dr. Guthnick, of the Berlin-Babelsberg Observatory, made a series of comparisons of Saturn and α Leonis by means of a photo-electric cell, correcting the differences of stellar magnitude for solar distance, phase angle, and ring effect, as far as this was possible. The planet still showed variations of brightness which Dr. Abbot† compared with the values of the solar constant determined at Calama, Chile. They agreed quantitatively in percentage with the variations of the solar constant, provided a time allowance was made to take account of the rotation of the sun from a position facing the earth to one facing Saturn. Dr. Abbot drew the conclusion, that‡ "it seemed, in short, as if rays of a certain intensity of radiation, going out from the sun, rotated along with it, and so affected the different planets in the order of their heliographic longitudes." One per cent. increase in solar radiation was found to accompany one per cent. increase of Saturn's brightness.

* Supplement to *Nature*, 1924 January 19, p. 106.

† *Annals of the Astrophysical Observatory of the Smithsonian Institution*, 4, 40-41 and 188-192.

‡ *Proc. Nat. Acad. Sci.*, No. 9, p. 356, October 1923.

The diminution in the solar constant seems to bear a similar connection with sunspots and with terrestrial magnetic storms as these do with each other: a connection which is real, but by no means rigid. The actions on the solar constant and on terrestrial magnetism appear to move with the same angular velocity as the nucleus of disturbance on the sun, and therefore both must be caused (directly or indirectly) by restricted streams issuing from the nucleus.

We do not know what relation the apex, which is drawn out into a rod-like ray, bears to the centre of a group or to any particular point in it that appears to be connected with it. We do not know whether the ray is always normal to the sun's surface, or whether its inclination to the surface is constant or changing. Therefore we cannot fix with certainty the length of time that has elapsed between the issue of the particles from the sun, and their action on the earth's magnetism. We do not know whether the particles are always expelled at the same speed, and whether their motion is constant or accelerated. For example, the difference in time between the 1920 February spot (9127) crossing the sun's central meridian and its magnetic storm was about 21 hours; the difference in March for the whole group 9143 and its magnetic storm was 18 hours and between its principal spot and leader and the storm was 1^{d} 11^{h}. There is some evidence that the motion of the constituent particles of the streams may be accelerated, for Mr. Evershed found in the case of certain prominences that * the most striking feature was the accelerating velocity with which the mass left the sun. It may, perhaps, be possible to determine this for the particles in the streams, if in the future Dr. Guthnick's values for the brightness are continued not only for Saturn but for other planets, both major and minor, and comparisons with the solar constant are multiplied.

We were moved to take up this inquiry by the paper of last December, where Dr. Jeans analysed the movements of bright markings which Mr. van Maanen had measured in spiral nebulæ. Dr. Jeans wrote †: "The general effect of the modified gravitational force appears to be one of seizing the particle and compelling it to revolve about the nucleus of the nebula with an angular velocity which approximates to that of the nucleus itself rather than to that demanded by the conservation of angular momentum. As a consequence, the particle moves as though it were repelled from the nucleus, and describes an orbit of ever-increasing radius with ever-increasing velocity. . . . It is quite clear that a force of the kind we have been considering, if it exists at all, can be of only insignificant magnitude in the solar system, or at least can differ only very slightly from the ordinary gravitational force."

We would like to point out that this action of the restricted solar streams taking place at the earth and at Saturn is similar to that which Dr. Jeans has described; and that it is appreciable within the solar system.

As one of the writers pointed out in 1904, this action by streams of particles differs from the radiation of light and heat by the sun in several

* *Astrophysical Journal*, **28**, No. 1, p. 80, July 1908.
† *M.N., R.A.S.*, **84**, No. 2, pp. 74–75, December 1923.

important particulars. The particles are expelled from the sun, not continuously, but spasmodically; not in all directions, but along definite and restricted lines; not from every part of the sun, but from certain restricted zones and from restricted regions in those zones. At some epochs the sun expels many particles from these restricted areas, at other times few particles. The effect on the members of the solar system must vary from time to time, but it will certainly be most unequal in the case of a planet with an orbit of large inclination to the ecliptic. We do not know how the effect may differ with the distance, but if Dr. Abbot's conclusion—that one per cent. of variation in the brightness of Saturn corresponds quantitatively to one per cent. variation in the solar constant—is confirmed, then the falling off in effectiveness is small within the solar system.

The solar action seems to be confined between certain zones of latitude; there is evidence to show that—at least sometimes—the action is mostly confined within certain longitudes. In the Second Report of the B.A.A. Section for the Observation of Auroræ, a table is given which shows clearly the connection between auroræ and the rotation of the sun between the years 1915–1919. Throughout these years, the auroræ showed a marked and steady preference for the first 13 days of each rotation, being sparse and sporadic in the last 14 days. During this period, the sun was, within its measure and degree, a one-armed spiral.

Observations of (*Ch.* 678) *U Persei*, 1919–24.

By A. N. Brown, M.A.

The following observations, 216 in number, continue the series published in *Monthly Notices*, 79, 513–519. Light curves have been drawn from them, from which the following tables have been compiled:—

	Maxima.			Minima.		
No.	Date.	Julian Date.	Mag.	Date.	Julian Date.	Mag.
12	1919 June 7	2117	7·9	1920 Jan. 16	2340	10·1
13	1920 Apr. 10	2425	7·9	1920 Dec. 7	2666	10·4
14	1921 Feb. 18	2739	7·9	1921 Oct. 18	2981	10·3
15	1922 Feb. 3	3089	7·7	1922 Sept. 5	3303	9·8
16	1923 Jan. 6	3426	7·9	1923 Aug. 8	3640	9·7
17	1923 Dec. 5	3759	8·4	1924 May 3	3909	9·4

The above phases are numbered in continuation of those previously observed by me. It will be noticed that the star has varied during these years from 7·7 – 8·4 mag. to 9·4 – 10·4 mag. (Hagen's scale). As

is shown in the following table, there are again very considerable deviations from the mean period (derived from maxima and minima), which for these five years is 320·7 days.

No.	(*a*).	(*b*).	(*c*).	(*d*).	No.	(*a*).	(*b*).	(*c*).	(*d*).
12	316	322	99	223	15	350	322	108	214
13	308	326	85	241	16	337	337	123	214
14	314	315	73	242	17	333	269	119	150

In the above table all the intervals are given in days, and the four columns show as follows:—(*a*) the interval between successive maxima; (*b*) the interval between successive minima; (*c*) the time taken for the rise from minimum to maximum; (*d*) the time taken for the fall from maximum to minimum.

It has again been extremely difficult to assign dates for the maxima of this star from the plotted observations. The rise, which during the period under review has always been more rapid than the fall, is usually interrupted by marked pauses; the light remains constant, or nearly so, for a considerable length of time; and the fall is gradual, and also often interrupted by prolonged pauses. It has always been far more easy to fix the dates of minima than of maxima.

My observations of U Persei from 1909 Sept. to 1922 Feb. were discussed by the late Mr. Edward Prentice in *Monthly Notices*, **82**, 413–415.

As previously, the majority of the comparisons printed below were made by the fractional method, but all of these have been reduced to steps, the value of a step in every case being ·1 magnitude. In order to distinguish fractional from step estimates, a semi-colon has been placed between each separate estimate. Thus on 1919 Oct. 14 a fractional followed by a step estimate is shown, on Nov. 18 there are two step estimates. Equal weight has been given to results obtained by each method. The comparison stars are numbered as in the catalogue of Hagen's *Atlas Stell. Var.*, series iii., and Hagen's magnitudes have again been adopted. These are admittedly inferior to the Harvard values, but have been retained upon the advice of Professor Turner, who has been kind enough to point out to me that a change from one system to the other might result in confusing anyone who may use these observations in the future. The Harvard values, however, for each comparison star employed are shown in the table which follows the observations. These are obtained from *Specola Astro. Vaticana*, xi., "Aggiunte al Catalogo dell' A.S.V.," p. 247, a copy of which was most kindly sent me by Father Hagen some years ago. He remarks that the magnitudes for comparison stars contained in series i., ii., iii. of the *A.S.V.* were computed from a linear formula for the purpose of engraving charts, no photographic magnitudes being available at that time, and that they were not intended for the reduction of observations.

The telescopes used were again 5-inch and 3-inch refractors, with powers varying from 25 to 120.

The Observations.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1919			
Apr. 24	2073	3 − 4, 7 + 1	8·5
May 12	2091	3 − 0·5	8·1
17	2096	3 − 0·5	8·1
22	2101	3 − 0·5	8·1
28	2107	3 + 1	7·9
June 3	2113	3 + 1	7·9
9	2119	3 + 1	7·9
16	2126	3 + 1	7·9
21	2131	3 − 1	8·1
26	2136	3 − 1·5	8·2
July 15	2155	3 − 5, 7 + 1·5	8·5
22	2162	7 + 1	8·5
30	2170	7 + 1	8·5
Aug. 6	2177	= 7	8·6
9	2180	= 7	8·6
15	2186	7 − 0·5	8·7
21	2192	7 − 1, 13 + 5	8·7
31	2202	7 − 3, 13 + 3	8·9
Sept. 6	2208	7 − 3, 13 + 3	8·9
9	2211	7 − 3, 13 + 2	9·0
17	2219	7 − 5, 13 + 1	9·1
26	2228	7 − 4, 13 + 2	9·0
Oct. 2	2234	7 − 5, 13 + 1	9·1
14	2246	7 − 5, 13 + 1 ; 16 + 5	9·0
26	2258	7 − 5, 13 + 1·5	9·1
31	2263	7 − 5, 13 + 1	9·1
Nov. 18	2281	16 + 1 ; 17 + 3	9·3
21	2284	= 16	9·4
25	2288	13 − 1·5, 16 + 1	9·3
Dec. 2	2295	16 − 2, 24 + 1	9·6
19	2312	17 − 1·5, 24 + 1	9·6
1920			
Jan. 1	2325	33 + 1	10·0
21	2345	19 − 5·5, 34 + 2	10·1
Feb. 6	2361	12 − 1·5, 16 + 1	9·3
11	2366	12 − 1, = 16	9·4

Date.	Julian Date.	Comparisons.	Ded. Mag.
1920			
Feb. 17	2372	13 − 1, 17 + 2	9·3
Mar. 7	2391	7 + 1	8·5
12	2396	3 − 4, 7 + 1·5	8·4
18	2402	3 + 1	7·9
22	2406	3 + 1·5	7·9
Apr. 12	2427	3 + 1	7·9
16	2431	3 + 1	7·9
21	2436	3 + 1	7·9
28	2443	3 + 1	7·9
May 3	2448	3 + 1	7·9
13	2458	3 − 1	8·1
19	2464	3 − 1, 7 + 4	8·2
23	2468	3 − 1·5, 7 + 4	8·2
June 8	2484	3 − 4·5, 7 + 1	8·5
19	2495	3 − 4·5, 7 + 1	8·5
July 14	2520	7 + 0·5	8·6
18	2524	7 − 1, 13 + 5	8·7
24	2530	7 − 4·5, 13 + 2	9·0
Aug. 9	2546	7 − 4·5, 13 + 1	9·1
15	2552	7 − 4·5, 13 + 1	9·1
19	2556	7 − 4·5, 13 + 1	9·1
26	2563	7 − 4 ; 13 + 1·5	9·0
Sept. 5	2573	7 − 5 ; 13 + 1	9·1
10	2578	7 − 4·5, 13 + 1	9·1
12	2580	7 − 4·5, 13 + 1 ; 16 + 2·5	9·1
16	2584	7 − 5, 13 + 1	9·1
19	2587	7 − 5, 13 + 1 ; 16 + 2 ; 17 + 3	9·2
Oct. 2	2600	13 + 1 ; 16 + 1·5 ; 17 + 3	9·2
6	2604	16 − 1 ; 17 + 0·5	9·5
11	2609	17 − 0·5	9·6
18	2616	= 24	9·7
23	2621	22 − 1·5, 33 + 3	9·8
Nov. 2	2631	33 − 1, 37 + 3	10·2
17	2646	33 − 1·5, 37 + 3 ; 34 + 1	10·2

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1920			
Nov. 21	2650	33 − 3, 37 + 1·5; 34 + 1	10·3
Dec. 1	2660	34 − 1, 37 + 1	10·4
28	2687	18 − 2, 34 + 5	9·8
1921			
Jan. 12	2702	7 − 1	8·7
31	2721	3 − 1, 7 + 5	8·1
Feb. 14	2735	3 + 1	7·9
20	2741	3 + 1	7·9
24	2745	3 + 1	7·9
26	2747	= 3	8·0
Mar. 2	2751	3 − 1·5	8·2
7	2756	3 − 1, 7 + 4	8·2
18	2767	3 − 1, 7 + 4	8·2
24	2773	3 − 1, 7 + 5	8·1
30	2779	3 − 5, 7 + 1	8·5
Apr. 4	2784	7 + 1	8·5
6	2786	7 + 1	8·5
12	2792	7 + 1	8·5
23	2803	7 + 1	8·5
28	2808	3 − 4, 7 + 1	8·5
30	2810	3 − 4, 7 + 1	8·5
May 8	2818	3 − 5, 7 + 1	8·5
22	2832	3 − 5, 7 + 0·5	8·5
June 6	2847	7 − 3, 13 + 3	8·9
11	2852	7 − 4, 13 + 2	9·0
28	2869	7 − 4, 13 + 1	9·1
July 5	2876	7 − 4, 13 + 1	9·1
8	2879	7 − 4, 13 + 1	9·1
12	2883	7 − 5, 13 + 1	9·1
20	2891	13 + 1·5	9·1
25	2896	7 − 5; 13 + 1	9·1
Aug. 9	2911	= 13	9·2
25	2927	17 − 1; = 24	9·7
Sept. 2	2935	17 + 0·5	9·5
5	2938	17 − 1, 24 + 1	9·6
7	2940	24 − 1, 29 + 1	9·9
Sept. 22	2955	23 − 1, 29 + 1	9·9
25	2958	23 − 2·5, 29 + 1	9·9
29	2962	29 − 1; 34 + 1	10·2
Oct. 4	2967	29 − 2, 34 + 1	10·2
14	2977	34 + 1	10·2
23	2986	= 34; 37 + 2	10·3
30	2993	23 − 3; 34 + 3	10·0
Nov. 6	3000	23 − 4, 34 + 2	10·1
10	3004	17 − 1; 23 + 1	9·6
23	3017	13 − 1; 16 + 1; 17 + 2	9·3
27	3021	12 + 2; 13 + 1	9·1
Dec. 4	3028	7 − 4; 13 + 2	9·0
20	3044	7 + 1	8·5
22	3046	3 − 5, 7 + 1	8·5
28	3052	3 − 3, 7 + 2	8·4
31	3055	3 − 2, 7 + 4	8·2
1922			
Jan. 10	3065	3 + 1	7·9
16	3071	3 + 1	7·9
Feb. 9	3095	2 − 6·5, 3 + 3	7·7
13	3099	2 − 8, 3 + 1·5	7·8
17	3103	3 + 1	7·9
24	3110	3 − 1·5	8·2
Mar. 2	3116	3 − 2, 7 + 4	8·2
7	3121	3 − 2, 7 + 4	8·2
12	3126	3 − 4·5, 7 + 2	8·4
20	3134	3 − 5, 7 + 1	8·5
27	3141	7 + 2	8·4
Apr. 1	3146	= 7	8·6
19	3164	7 − 2, 13 + 4	8·8
22	3167	7 − 3, 13 + 3	8·9
May 4	3179	7 − 5, 13 + 1·5; 16 + 4	9·0
19	3194	13 + 1; 16 + 3; 17 + 4	9·1
27	3202	7 − 4, 13 + 1; 16 + 3	9·1

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1922			
June 1	3207	7 − 5, 13 + 1·5	9·1
4	3210	7 − 4, 13 + 1; 16 + 3	9·1
July 18	3254	13 − 1; 16 + 1; 17 + 2	9·3
30	3266	13 + 1; 16 + 2	9·2
Aug. 20	3287	16 − 0·5, 17 + 0·5	9·5
23	3290	17 − 1, 24 + 1	9·6
31	3298	24 − 1, 29 + 2	9·8
Sept. 11	3309	24 − 1·5, 29 + 2	9·8
20	3318	13 − 2, 17 + 1	9·4
24	3322	13 − 2, 17 + 1	9·4
29	3327	13 − 1·5, 17 + 1	9·4
Oct. 6	3334	13 − 1, 17 + 2	9·3
12	3340	13 + 1	9·1
14	3342	13 + 1·5	9·1
22	3350	7 − 5, 13 + 1·5	9·1
28	3356	7 − 4, 13 + 1	9·1
Nov. 15	3374	7 − 1, 13 + 4	8·8
24	3383	3 − 4, 7 + 1	8·5
27	3386	3 − 4, 7 + 2	8·4
Dec. 8	3397	3 − 1; 7 + 5	8·1
19	3408	3 − 1·5, 7 + 5	8·1
24	3413	3 − 1, 7 + 4	8·2
1923			
Jan. 8	3428	3 + 1	7·9
14	3434	3 − 1, 7 + 5	8·1
20	3440	3 − 1·5, 7 + 5	8·1
24	3444	3 − 1·5, 7 + 5	8·1
Feb. 4	3455	3 − 1, 7 + 4	8·2
Mar. 4	3483	= 7	8·6
17	3496	7 − 4, 13 + 2	9·0
Apr. 8	3518	7 − 4; 13 + 2	9·0
13	3523	7 − 5, 13 + 1·5	9·1
20	3530	13 + 1	9·1
26	3536	7 − 5; 13 + 1	9·1
May 17	3557	7 − 4, 13 + 1	9·1
1923			
May 21	3561	7 − 5, 13 + 1·5	9·1
July 4	3605	13 − 1, 17 + 1	9·4
21	3622	13 − 2, 17 + 1	9·4
25	3626	= 17	9·5
Aug. 4	3636	= 17	9·5
8	3640	17 − 2; 19 − 1; = 21	9·7
11	3643	17 − 1·5, 29 + 4	9·6
15	3647	= 17	9·5
30	3662	7 − 5, 13 + 1·5	9·1
Sept. 5	3668	7 − 4·5, 13 + 2	9·0
8	3671	7 − 4, 13 + 1	9·1
15	3678	7 − 4, 13 + 2	9·0
23	3686	= 7	8·6
Oct. 4	3697	3 − 4, 7 + 1	8·5
14	3707	3 − 4, 7 + 1	8·5
22	3715	3 − 4, 7 + 2	8·4
28	3721	3 − 4; 7 + 2	8·4
Nov. 3	3727	3 − 4, 7 + 1	8·5
7	3731	3 − 3; 6 + 2; 7 + 3	8·3
14	3738	3 − 4; 7 + 2	8·4
30	3754	3 − 4; 6 + 2; 7 + 2	8·4
Dec. 4	3758	3 − 5; 6 + 2; 7 + 2	8·4
15	3769	3 − 4·5, 7 + 2	8·4
23	3777	3 − 4; 7 + 3	8·4
1924			
Jan. 2	3787	3 − 4; 6 + 2; 7 + 3	8·3
6	3791	3 − 4; 7 + 3	8·4
25	3810	3 − 4; 7 + 3	8·4
29	3814	3 − 5; 7 + 2	8·5
Feb. 1	3817	3 − 5; 7 + 2	8·5
16	3832	3 − 5; 7 + 2	8·5
24	3840	= 7	8·6
Mar. 2	3847	7 − 1·5	8·8
4	3849	7 − 1; 13 + 5	8·7
7	3852	7 − 1, 13 + 5	8·7
11	3856	7 − 1, 13 + 5	8·7

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1924			
Mar. 19	3864	7 − 3·5, 13 + 2	9·0
29	3874	13 + 1	9·1
Apr. 1	3877	7 − 5, 13 + 1	9·1
4	3880	7 − 5; 13 + 1	9·1
7	3883	13 + 0·5	9·2
20	3896	7 − 5, 13 + 1	9·1
1924			
May 1	3907	13 − 2, 17 + 1	9·4
5	3911	13 − 1, 17 + 1	9·4
16	3922	7 − 5; 13 + 1; 17 + 3	9·1
27	3933	7 − 5; 13 + 1	9·1

The Comparison Stars Used.

Hagen's Number.	Hagen's Mag.	H.P. Mag.	Hagen's Number.	Hagen's Mag.	H.P. Mag.
2	*7·0	7·5	19	9·6	9·9
3	8·0	7·3	21	9·7	10·0
6	8·5	8·3	22	9·7	10·0
7	8·6	8·6	23	9·7	10·0
12	9·2	9·5	24	9·7	10·0
13	9·2	9·5	29	10·0	10·2
16	9·4	9·7	33	10·1	10·4
17	9·5	9·8	34	10·3	10·5
18	9·5	9·8	37	10·5	10·8

Brackenhurst,
Bucklebury Common, Berks:
1924 *June* 4.

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l Carinæ. (R.A. $9^h\ 42^m\ 30^s$, Decl. − 62° 2·8′ (1900).)

By A. W. J. Cousins.

(*Communicated by Dr. A. W. Roberts.*)

History.—The variability of the star *l* Carinæ was discovered by Gould, who made a large number of observations to determine the nature of its variation. He deduced a period of about $31\frac{1}{4}$ days and a range from $2^m\cdot7$ to $5^m\cdot2$, but acknowledged that the results were not very satisfactory. (U.A.)

An important series of observations was made by Dr. A. W. Roberts during the years 1891–99, and the results were published at its completion (*Astr. Journ.*, 491, 492 (1901).

Dr. R. T. A. Innes has also published a few observations made during the years 1896–97 (*Journal B.A.A.*, 8, No. 5, p. 228).

Apart from these observations, very little attention has been paid to this most interesting star.

* The B.D. value as given in Hagen's Catalogue.

Observations.—The present series of observations covers the years 1920–23. Practically all observations were made with the naked eye, but on a few occasions a pair of binoculars was used.

The step method has been employed throughout. The value of the step used is so close to $0^{m}\cdot1$, that for all purposes of reduction it may be accepted as such. The deduced magnitude has been determined by taking the arithmetic mean of the individual estimates.

The magnitudes of the comparison stars have been obtained from the Harvard Catalogues.

The deduced magnitudes were then plotted consecutively and the dates of maximum and minimum read off. These are given in Table I.

TABLE I.

Dates of Maxima and Minima.

Maxima.				Minima.			
Observed. J.D. 242+	Computed. J.D. 242+	O–C. d.	Mag.	Observed. J.D. 242+	Computed. J.D. 242+	O–C. d.	Mag.
				2386	2389	−3	3·9
2401	2400	+1	3·2	2420	2424	−4	3·8
2438	2436	+2					
2472	2471	+1					
...	...	...	...	2527	2531	−4	4·0
2540	2543	−3	3·1				
				2743	2744	−1	4·0
2756	2756	0	3·3	2780	2779	+1	4·0
2789	2791	−2	3·4	2813	2815	−2	3·9
2826	2827	−1	3·4	2853	2850	+3	4·0?
2865	2862	+3	3·3	2889	2886	+3	4·3
2897	2897	0	3·5	...	...	...	<4·2
3043	3040	+3	3·4	3066	3064	+2	4·1
3073	3075	−2	3·4	3102	3099	+3	4·1
3110	3111	−1					
3146?	3147	−1	...	3173	3170	+3	4·4
3182	3182	0	3·3	3208	3206	+2	4·4
3221?	3217	(+4)	...	3241	3241	0	4·4
3253	3253	0	3·4				
				3457	3455	+2	4·0
3466	3466	0	3·4	3492	3490	+2	4·1
3501	3501	0	3·4	3528	3525	+3	4·2
3537	3537	0	3·3	3564	3561	+3	4·2
3573	3572	+1	3·3	3597	3596	+1	4·1
3609	3608	+1	3·3	3635	3632	+3	4·2
3644	3644	0	3·3				

These dates were then reduced to a mean epoch for each year and a mean was taken (A).

The observations were next reduced to this mean date and plotted together. A mean curve was then drawn through these points and a second mean epoch determined (B).

These two dates combined gave the adopted mean epoch for each year, and from these again was determined the mean maximum and minimum for the series. (Table II.)

TABLE II.

Mean Maxima and Minima.

	Maxima.			
A.	1920.	1921.	1922.	1923.
	J.D. 242+	J.D. 242+	J.D. 242+	J.D. 242+
A.	2472±1	2862±2	3182±1	3537±·5
B.	2472	2865	3183	3537·5
Mean	2472±1	2863±2	3182·5±1	3537·3±·5
	2863	2863	2862·6	2862·4
	Mean from above 2862·7±·3			
	graphically 2863 Mag. 3·3			

	Minima.			
A.	2492±1	2886±2	3208±1	3563±1
B.	2493	2887	3206	3562
Mean	2492·5±1	2886·5±2	3207±1	3563±1
	2883·5	2886·5	2887	2888
	Mean from above 2886±1			
	graphically 2887 Mag. 4·1			

All the observations were then plotted together and a mean curve for the series drawn. This gave additional values for the mean maximum and mean minimum, and also a curve for harmonic analysis.

Light Curve.—A harmonic analysis has been made of the mean light curve embracing three harmonic terms:

$$\begin{aligned} M = 3{\cdot}825 &+ {\cdot}315\cos(10^\circ{\cdot}1t + 155^\circ) \\ &+ {\cdot}102\cos(20^\circ{\cdot}3t + 191^\circ) \\ &+ {\cdot}051\cos(30^\circ{\cdot}4t + 237^\circ) \end{aligned}$$

where t is reckoned in days.

This gives a fair idea of the general character of the curve but does not show the slight suggestion of a secondary maximum.

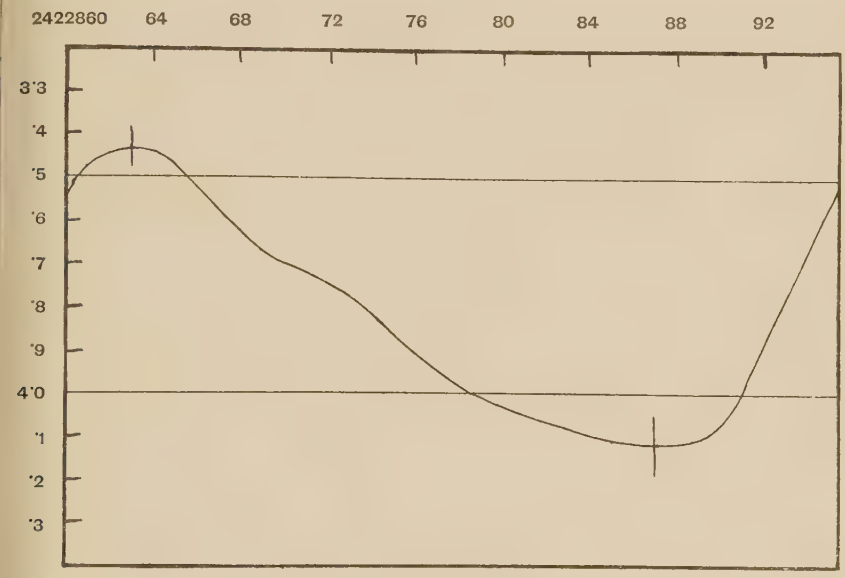
This curve is very typical of one type of curve, but another type has also been observed. Sometimes the drop after maximum is at

first very rapid, but slackens afterwards. In this respect it tends more to the type of variation found in clusters.

Generally speaking, the secondary maximum is not evident in the individual curves, but it has occasionally been observed.

Throughout the series the maximum magnitude has remained very constant, while the minimum has shown considerable variation. Minima as bright as $3^m{\cdot}8$ or as faint as $4^m{\cdot}4$ have occasionally been observed, but the mean value is near $4^m{\cdot}1$.

The differences between the dates of observed maxima and those



Light Curve of *l* Carinæ.

computed from the mean maximum are only small. The larger values in the case of the minima result from the difficulty of determining the dates correctly from so flat a curve.

Period.—The series extends over too short a time to give a good value for the length of the period. A comparison has, however, been made with earlier maxima.

From this series alone :		$35^d{\cdot}57 \pm {\cdot}05$	1920–1923
Comparing with :	Roberts	$35^d{\cdot}537 \pm {\cdot}002$	1900–1921
	Innes	$35^d{\cdot}542 \pm {\cdot}004$	
	Gould	$35^d{\cdot}532 \pm {\cdot}002$	1871–1921
Dr. Roberts himself gives		$35^d{\cdot}523$	1891–1899
and Innes		$35^d{\cdot}5236$	

These results would suggest that the period is slowly lengthening.

SS Aurigæ. By P. M. Ryves.

This star was announced as variable 31·1907 in *Ast. Nach.*, **174**, 363; max. mag. $8\frac{1}{2}$, min. < 14; and was subsequently reported by Hartwig to exhibit fluctuations similar to those of *U Geminorum*.* The Harvard number is 060547, final designation *SS Aurigæ*.

During the fifty years following the discovery (by Hind in 1855) of *U Geminorum*, no other star had been found to show the same characteristics. *SS Cygni* (1896) was at first assigned to the *U Geminorum* class, but after the very irregular behaviour of that star in 1908, it was pointed out by the writer † that *SS Cygni* could no longer be considered as a true *U Geminid*, although it presents several analogies, a view which subsequent experience has confirmed.

The discovery of a new and bright variable, possibly of the same class, and more favourably placed in the sky for continued observation than the type-star, was thus a matter of considerable interest.

Observations were commenced at Zaragoza in 1910, but it soon became apparent that the maximum brightness had been much overrated (it scarcely exceeds the 11th mag.), and that the star would prove a faint object for small telescopes. However, the observations were continued regularly for a time, and *SS* was seen at or near 12 maxima. The first maximum was observed in 1910 December, three were caught in 1911, four in 1912, three in 1913, and one in 1914. As very few observations have been published in Europe, and there is not much material available for the study of the light-changes of this star during the years following its discovery, some details of the above series may be of sufficient interest to warrant publication. Observations of three more recent maxima are appended.

The Comparison Stars used were selected from the list given in *Harvard Circular*, No. 138. It was noticed that star *r* (12·64) generally appeared brighter than *q* (12·42); *k* (10·79) distinctly fainter than *g* (10·76); and *s* (12·98) sometimes nearly equal to *t* (13·26), and these small discrepancies are to some extent confirmed by the details contained in the table of revised magnitudes recently published by the Leander McCormick Observatory.‡

Particulars of the stars used and some fainter ones are contained in the adjoined table which is based upon information from the above sources. The designations are those of H.C.O. Down to *q*, following the lettering, the values in the fourth column are the means of the photometer measures of Harvard (12-inch meridian photometer), and McCormick (photometer attached to the 26-inch equatorial) except *g* which is Harvard alone; the remainder are McCormick alone. The adopted magnitude is that given by McCormick, being a combination of the results of the two observatories.

* See *Harvard Bulletin*, 324.
† *Anales de la Facultad de Ciencias de Zaragoza*, 1908 December.
‡ *A.J.*, 779.

TABLE I.

Comparison Stars for SS Aurigæ.

	R.A. 1900.	Dec. 1900.	Phtm.	Adopted Mag.
	h m s	° ′		
f	6 6 12·0	+47 35·2	10·51	10·52
g	6 8·2	42·3	10·78	10·77
k	5 48·9	47·7	10·93	10·82
**l*	6 30·9	49·2	10·92	11·00
m	5 30·2	37·4	11·21	11·20
n	7 4·9	49·8	11·53	11·46
**o*	5 21·0	39·8	11·37	11·51
p	5 48·1	32·1	11·70	11·78
r	5 14·9	48·1	12·05	12·37
q	6 9·3	46·2	12·46	12·46
s	5 18·3	41·2	13·05	13·02
t	5 27·0	44·1	13·11	13·21
u	5 35·7	44·1	13·79	13·78
w	5 39·0	46·2	13·94	13·94
x	5 54·6	45·7	14·64	14·54
y	5 58·1	45·4	...	14·83
z	5 55·0	45·3	...	15·18
a	5 46·5	45·2	...	15·8
SS	6 5 48·6	+47 45·7	...	...

Note.—I feel rather suspicious of stars *l* and *o*; the magnitudes do not seem quite satisfactory, and it is possible that there may be slight variability; both are reddish, and this alone is sufficient reason for avoiding them as comparison stars for *SS*.

The Observations were made by the method of Argelander, using 3-inch and 5-inch refractors. With such apertures the majority of the observations must necessarily be negative, marking an upper limit, since, except during its irregularly periodic outbursts, *SS Aurigæ* is beyond the reach of all but large instruments. Details of the positive observations are given in Table II., comprising calendar date (astronomical); day of the Julian period (G.M.T.); deduced magnitude, taking values for comparison stars from the last column of Table I.; and weight on a scale of 1 – 5, 5 = best.

[TABLE II.

TABLE II.

Maxima of SS Aurigæ.

1910–11			241–242.	Mag.	W.
	Dec.	20	9026·25	<13·15	4
		25	9031·28	<11·8	3
		26	9032·40	11·0	3
		28	9034·28	10·80	3
		29	9035·25	10·67	2
		30	9036·30	10·65	3
(1)		31	9037·30	10·67	4
	Jan.	2	9039·56	11·16	2
		3	9040·53	11·20	3
		4	9041·27	11·61	2
		5	9042·53	12·46	2
		7	9044·56	<13·0	
	May	13	9170·36	<11·0	3
		17	9174·38	10·74	3
		21	9178·37	10·72	3
(2)		22	9179·38	10·80	3
		23	9180·43	11·00	1
		25	9182·41	11·36	2
	Aug.	13	9262·60	<11·3	3
		17	9266·61	11·00	3
		19	9268·57	10·82	3
		20	9269·57	10·82	3
		21	9270·55	10·72	3
(3)		22	9271·61	10·90	3
		23	9272·58	11·20	3
		24	9273·64	11·78	3
		25	9274·65	12·27	3
		26	9275·59	13·07	3
		27	9276·64	14·0	3
	Nov.	21	9362·50	<12·45	3
		22	9363·53	<12·0	3
		27	9368·60	11·07	3
		29	9370·60	10·67	4
(4)		30	9371·57	10·67	3
	Dec.	5	9376·50	10·82	2
		7	9378·49	<11·2	2
		10	9381·51	<12·1	4

1912			241–242.	Mag.	W.
	Feb.	16	9449·31	<13·4	3
		18	9451·54	<11·8	3
		19	9452·52	11·0	4
		20	9453·32	10·82	2
(5)		20	9453·56	10·85	3
		21	9454·49	10·85	4
		23	9456·52	11·30	2
		25	9458·59	11·97	3
	April	6	9499·34	<13·1	4
		8	9501·34	<12·0	4
		9	9502·34	11·10	3
		10	9503·34	10·85	4
		13	9506·37	11·08	2
(6)		14	9507·38	11·20	2
		15	9508·36	11·00	3
		17	9510·37	11·30	4
		18	9511·34	11·55	4
		19	9512·40	12·36	3
	Sept.	27	9673·58	<11·45	3
		28	9674·60	11·07	3
	Oct.	2	9678·62	10·72	3
		3	9679·61	10·70	3
		4	9680·58	10·87	3
(7)		5	9681·54	10·93	4
		7	9683·51	11·62	2
		8	9684·59	11·68	3
		9	9685·58	12·25	3
		10	9686·59	12·75	2
		12	9688·60	13·8	1
	Nov.	13	9720·40	<13·2	3
		14	9721·40	<11·8	3
		16	9723·57	<11·5	3
(7½)		18	9725·60	11·35	2
		19	9726·55	11·27	4
		20	9727·38	11·62	2
		22	9729·63	<11·8	3

TABLE II—*continued.*

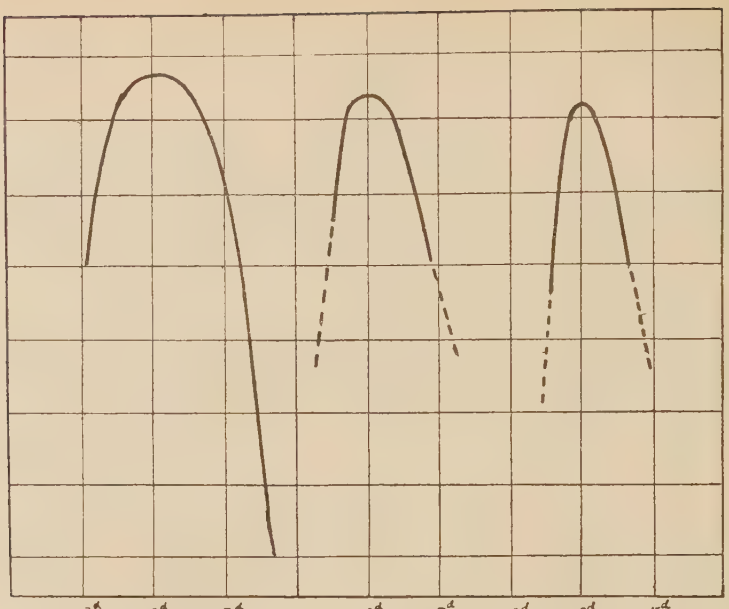
			241–242.	Mag.	W.
1913					
	Jan.	18	9786·72	<13·0	3
(8)		25	9793·28	11·2	3
		27	9795·29	12·65	3
	Aug.	29	0009·58	<13·2	3
		31	0011·57	11·40	2
	Sept.	1	0012·62	10·72	3
(9)		2	0013·63	10·85	3
		4	0015·61	11·78	3
		5	0017·62	12·17	4
1913					
	Oct.	29	0070·58	10·85	3
(10)		31	0072·56	11·00	2
	Nov.	2	0074·62	11·70	2
1914					
	Apr.	20	0243·42	11·25	2
(11)		22	0245·45	11·20	2
		27	0250·41	<11·0	3

New Series.

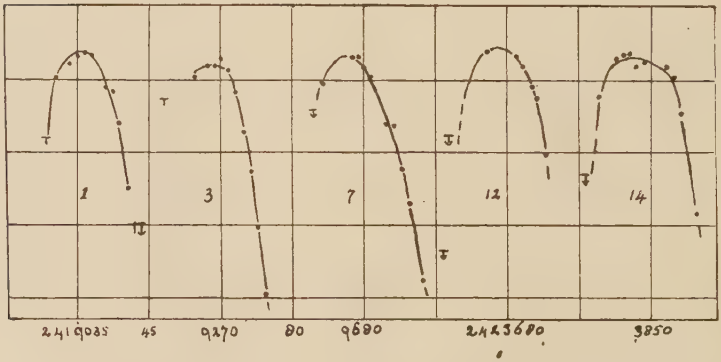
			242.	Mag.	W.
1923					
	Sept.	8	3671·63	<13·4	3
		9	3672·60	<11·8	3
		10	3673·65	<11·7	3
		14	3677·60	10·62	3
		18	3681·58	10·67	3
(12)		19	3682·54	10·84	4
		20	3683·62	11·15	3
		21	3684·63	11·27	3
		22	3685·67	12·08	3
		23	3686·71	(11·8)	1
		24	3687·65	<11·8	3
	Nov.	11	3735·43	<13·4	3
		12	3736·46	<12·2	3
		13	3737·45	11·14	2
		14	3738·43	11·03	2
(13)		15	3739·51	11·16	4
		16	3740·57	11·42	3
		17	3741·46	12·02	4
		18	3742·42	<11·9	3
1924					
	Feb.	25	3841·43	<12·35	3
		27	3843·53	<11·3	3
		29	3845·46	10·77	3
	Mar.	1	3846·52	10·70	3
		2	3847·45	10·70	3
(14)		3	3848·40	10·82	3
		4	3849·50	10·80	2
		7	3852·52	10·82	4
		8	3853·47	11·00	4
		9	3854·53	11·47	3
		11	3856·59	12·91	2

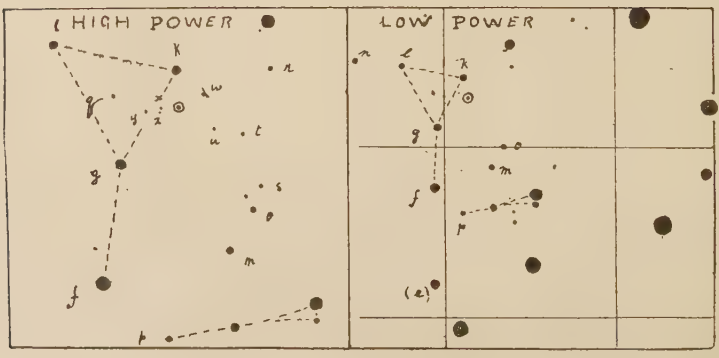
There was also a rather doubtful entry, 1924 January 27 (2423813) mag. 12·20, which, however, may have been intended for an upper limit estimation.

No attempt has been made to draw general conclusions from these observations, which must be considered in conjunction with those made elsewhere. The following points, however, may be briefly referred to.



THREE TYPICAL MAXIMA





SS Aurigæ.

The observations do not establish a regular alternation of *long* with *short* maxima, a characteristic of the variation of *U Geminorum*, but they do not exclude the possibility of such an alternation. Abnormal primary maxima (slow increase), such as frequently occur with *SS Cygni*, were not observed ; neither is there definite evidence of any secondary maximum having taken place, though it should be noted that with the small light-grasp available, these would be likely to escape the observer. Of the three typical curves shown in the diagram, that on the left, based upon 61 observations, and that on the right (12 observations), may be taken as exemplifying *long* and *short* maxima respectively, whilst the middle curve, which is Max. No. 5, appears to represent an intermediate type. At the short maxima *SS Aurigæ* remained above the 12th mag. for 5 days and at the long maxima 11 days. At long maxima the curve is more rounded than is the case with *SS Cygni*, and both in these and in the short maxima the light-curve of *SS Aurigæ* appears to resemble that of the type-star *U Geminorum*. Max. No. 14 was flatter than the rest, and No. 6 seems to have been unlike any of the others, but the observations were not very satisfactory. It is possible that no two maxima are exactly alike. The very regular descent from 11th to 14th mag. is best seen in No. 3.

Both in the amplitude of the variation and in the character of the light-curve, *SS Aurigæ* appears to resemble *U Geminorum* more closely than any other known variable ; in fact, no other star has yet been found to conform strictly to the type of variation exhibited by Hind's discovery of seventy years ago.

It is desirable that this star should be followed closely with instruments of considerable light-grasp.

Zaragoza, Spain :
1924 *June* 7.

Baxendell's Observations of β Lyræ. Edited by Mary A. Blagg.*

These observations of β Lyræ, made by J. Baxendell, senior, between the years 1840–77 (inc.), have been treated on much the same lines as those of the long period variables already discussed (*M.N.*, 73 to 78), with such modifications as the different conditions necessitated.

There are more than 1200 observations, scattered over most of the thirty-eight years, but falling mainly between 1848 Sept. and the end of 1861. They were taken with the naked eye, with the exception of a very few which are indicated.

In this case there are no difficulties in identifying the comparison stars, as these are all well known and are called by their ordinary appellations. Table I. gives the list of them, with their magnitudes.

* Professor Turner very kindly allowed me to send him my first draft of this paper, and I am deeply indebted to him for his advice, especially in regard to the improvement and simplification of some of the tables. In Table VII. the columns C_1 and CF (though the actual figures are my own) owe their form entirely to his detailed suggestions.

TABLE I.

Draper Catalogue.

Star's Name.	Vis. Mag.	Ph. Mag.	Spect.	Adopted Mag.
γ Lyræ	3·30	3·30	Aop	3·30
δ_1 ,,	5·51	5·34	B3	4·15
δ_2 ,,	4·52	5·87	Mb	
ζ ,,	4·29	4·37	A3	4·10
	5·87	5·95	A3	
κ ,,	4·34	5·34	Ko	4·30
δ Herculis	3·16	3·22	A2	3·20
μ ,,	3·48	4·26	G5	3·48
ξ ,,	3·82	4·82	Ko	3·76
o ,,	3·83	3·83	Ao	3·88
θ ,,	3·99	4·99	Ko	3·92
δ Cygni	2·97	...	...	(2·97)

In what follows, the decimal point is omitted and magnitudes are expressed in units of 0·01. θ Herculis was used for comparison in 1848 and 1849 only. The results were discordant, which was perhaps the reason for discarding it. δ Cygni was used only once: the comparison has not been used.

In adopting magnitudes (last column), attention has been paid to Baxendell's personal peculiarities. His observations show that he saw the Ko stars brighter than R.H.P., while o Herculis appeared to him fainter.

Four different methods of recording comparisons were employed.

(1) From 1840 to 1856 (inc.) *symbols* were chiefly used. These symbols are the same in form as those used by Herschel, but they do not represent the same quantities as with him. Their values were very fully investigated in the course of the former work on Baxendell's observations, and it was found that they varied with the magnitude of the stars, *or* with the means of observation used—it is hard to decide which. For stars observed with the naked eye (down to about 3·8 m.) the following values were obtained, representing approximately hundredths of a magnitude:—

TABLE II.

Symbol.	Value.	Symbol.	Value.	Symbol.	Value.
.	5	⊥ or ⊤	18	_,	35
;	9	–	23	_ ⊥	45
,	13	=,	27	— —	55

(2) From 1851 onwards, especially in 1855–7, a proportional system of comparison was sometimes used.

(3) In 1841, for two months, vertical strokes were used. These have been taken to have the same meaning as the *figures* in use later on, and the results support this view well enough; but there is no actual proof of its correctness, so these comparisons have not been used in Table III.

(4) From 1857 plain figures were mainly used, at first with the decimal point (afterwards discarded) showing that they were intended for fractions of a magnitude.

These "steps" were found, in the previous work, to need reduction in order to make them (approximately) equal to tenths of a magnitude. Table III. shows that the symbol values need a similar reduction in this case, and this has been applied throughout in Table IV. to both symbols and "steps."

Table III.

Baxendell's differences between his comparison stars expressed first in his own units and finally in units of 0·01 *mag. From* 1840–56 *the comparisons are interpreted from his symbols. The figures in brackets show numbers of observations.*

Pair of Stars.	1840–4.	1848–56.	1857–77.	General Mean.	
				Bax. Units.	0·01 Mag.
348–330	29 (3)	25 (69)	19 (17)	24 (89)	21
376–348	40 (5)	30 (129)	28 (4)	30 (138)	27
388–376	36 (3)	23 (97)	...	24 (100)	21
410–388	25 (6)	34 (99)	32 (47)	33 (152)	29
415–410	7 (6)	8 (4)	...	8 (10)	6
430–410	0 (1)	21 (19)	...	20 (20)	18
348–320	...	31 (14)	20 (1)	30 (15)	27
410–330	76 (2)	84 (2)	87 (13)	85 (17)	76
392–388	...	6 (40)	...	6 (40)	5
410–392	...	38 (23)	...	38 (23)	33
392–376	...	26 (21)	...	26 (21)	24
376–330	58 (2)	40 (10)	...	43 (12)	38

There are two possible ways of treating the observations when the variable is compared with two or more stars. The comparisons may be regarded as independent observations, and the arithmetic mean taken; or each pair may be considered as *one* observation, and the value taken proportionally. The first was the method adopted with the variables previously discussed; but with β Lyræ it would sometimes give very anomalous results, and the proportional plan has therefore been used when the variable is *between* the two stars in magnitude. When it is brighter or fainter than both, and when it is compared with more than two stars, the comparisons have been treated as independent ones. When the quantities are small or not very discordant, there is, of course, little or no difference between the results of the two methods.

TABLE IV.

Baxendell's Observations of β Lyræ.

[Until the end of 1858 Manchester mean local time was used. The required correction (+9 mins.) to convert it into G.M.T. has been applied to the Julian dates, but not to the others.]

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1840 Feb. d h m			
25 12 0	3161·51	330−5, 410−31 "stars v. low"	(357(?))
29 12 45	3165·54	330+24, 415−31, 410−31, 348−8	364(?)
Mar.			
5 12 30	3170·53	415−5	410
Apr.			
25 11 0	3221·47	410−5	405
26 11 15	3222·48	410−20, 388−8	385
30 11 15	3226·48	376−5, 410−31	375
14 25	3226·61	376−5	371
May			
1 10 35	3227·44	388−12, 376+20	386
2 11 0	3228·47	388−12, 410−24	381
14 30	3228·61	410−31, 376+24, 388−16	384
3 10 40	3229·45	376+5, 388−20	378
Aug.			
7 10 40	3325·45	410−16, 388+5 "β is of about the same brightness as at the same hour last night"	394
8 10 40	3326·45	348+8, 376−20	356
Sept.			
1 9 15	3350·39	410−5, 415−16	402
4 10 15	3353·43	330+20, 376−20, 348+12	355
18 9 0	3367·38	348+20, 376−20, 410−31	368
Oct.			
25 7 15	3404·30	376+5	381
1841 Sept.			
12 8 50	3726·38	376+0, 388−18, 410−26	377
1841 Sept. d h m			
17 10 5	3731·42	376+26, 410−18, 388+0	394
18 9 25	3732·40	348+26, 388−26, 376+0	371
19 10 30	3733·44	348+18, 376−9	367
22 11 10	3736·47	376+9, 388−9	382
23 10 15	3737·43	388+26, 410−18	401
24 10 35	3738·45	388+9, 410−26	394
29 8 30	3743·36	376+0, 388−9, 410−26	380
Oct.			
2 11 15	3746·48	330+26, 410−35, 388−26, 348+0	360
3 7 40	3747·33	348+18, 376−18, 388−26	362
5 9 0	3749·38	348+26, 388−18, 376+0	374
16 11 20	3760·48	330+26, 410−35	364
19 6 55	3763·29	388+35, 415−9, 410−18	407
Nov.			
14 6 40	3789·28	410−5, 388+50	408
Dec.			
1 7 0	3806·30	330+31, 388−31, 410−50	359
1842 May			
2 10 10	3958·43	376+8, 415−31, 410−31, 430−31	386
5 11 25	3961·48	330+31, 348−5, 376−31	350
June			
9 11 0	3996·47	410−16	394
Sept.			
29 9 55	4108·42	348+31, 376−20	367

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1842 Oct. d h m			
10 10 10	4119·43	330+50, 415−31	382
1844 Aug.			
4 11 0	4783·47	410−12, 415−31	[391(?)]
1848 Sept.			
4 9 28	6275·40	376+½(388−376)	382
10 11 0	6281·47	376+½(388−376) "rather uncertain"	382(?)
11 12 0	6282·51	376+½(388−376)	382
16 9 15	6287·39	330+31, 376+20, 388−12, 392−12	378
19 8 20	6290·35	330+31, 388−20, 376−20	362
22 11 0	6293·47	388+50, 410−20, 430−20, 392+31	415
23 11 35	6294·49	410−50	360(?)
24 11 0	6295·47	410−40	370(?)
26 11 15	6297·49	410−50	360
27 10 5	6298·43	388+5, 392−20	389
Oct.			
5 9 20	6306·40	388+50, 392+31, 430−20, 410−16	416
11 0	6306·47	392+31, 410−20	403(?)
7 7 45	6308·33	376+5, 388−20, 392−31	370
8 8 20	6309·35	330+31, 376−20, 388−31	358
9 8 55	6310·38	376−24	352
10 7 5	6311·30	348+31, 376−5, 388−20	373
7 50	6311·33	376+12, 388−16, 392−20	377
9 0	6311·38	376+16, 388−12, 392−16	381
11 30	6311·49	376+16, 392−20	382
11 12 20	6312·52	392−8 "stars v. low"	384
14 7 0	6315·30	376−20, 388−31	356
9 35	6315·41	376−20, 388−31	356
15 9 35	6316·41	376−20, 388−50	(347(?)
17 8 35	6318·37	392+31, 410−20	403
9 45	6318·41	388+31, 392+31, 410−20	411
18 6 25	6319·27	410+8, 392+31, 388+50	426
7 20	6319·31	410+12, 430 20	418
8 55	6319·38	410−16, 430−31	396
10 50	6319·46	410−16, 430−31	396
19 6 15	6320·27	376+8, 388−12, 392−20	377
20 6 45	6321·29	348+20, 376−12	366
9 0	6321·38	376+12, 388−24 [? 376−12]	(380 ?) [? 364]
22 6 30	6323·28	348+20, 376+5, 388−20	372
6 50	6323·29	376−8	368
25 8 50	6326·37	392−20,* 376+12, 388−12	379
26 10 50	6327·46	392−31	361
27 8 0	6328·34	348+12, 376−20	358
28 7 15	6329·31	348+12, 376−20	358
29 6 40	6330·28	376+12, 388−12	382
30 5 55	6331·25	392+24,* 388+31, 410−20	406
10 30	6331·45	410−20 "but low"	390(?)
Nov.			
1 8 15	6333·35	"β in phase of greatest lightness, but comparison stars obscured by clouds"	

* Half weight is given to these observations.

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1848 Nov. d h m			
3 7 40	6335·33	348+24, 376−16	366
4 6 35	6336·28	348+31, 376−8, 388−31, 392−31	366
5 7 0	6337·30	392+5,* 388−8, 376+16	388
6 7 10	6338·30	392+12,* 388+16	404
8 5	6338·34	392+20,* 388+20	409
7 8 0	6339·34	388−24, 392−24,* 376−5	368
9 6 10	6341·26	348+20, 376−20, 330+20	358
10 6 30	6342·27	330+20, 376−20, 388−40, 392−40	352
8 30	6342·36	376−12, 388−50, 392−50, 330+20	348
12 9 20	6344·39	392*+20, 410−12 "low"	403(?)
14 7 10	6346·30	348+31, 376−8, 388−20, 392−20	372
17 6 35	6349·28	388−31, 392−31, 376+20 †	371
18 9 20	6350·39	392−20 "but v. low"	372(?)
21 5 35	6353·24	376−20, 388−31, 392−31	358
23 6 15	6355·27	392−31	361
30 5 15	6362·22	330+50	380
Dec.			
4 5 5	6366·22	376−5	371
5 6 20	6367·27	348+12, 376−16, 330+20	357
9 5 15	6371·22	388+24, 392+16, 410−24	402
13 6 5	6375·26	392−24, 330+24	361
17 5 20	6379·23	392−31, 330+20	355
20 6 20	6382·27	388+12, 392+5	398
21 5 45	6383·25	388+31, 392+24, 410−12	411
22 5 45	6384·25	388+16, 392+12, 410−24	398
23 7 20	6385·31	392−31, 330+24	358
28 6 5	6390·26	392−5, 410−31 "rather low, and atmosphere foggy"	383(?
1849 Jan.			
11 5 40	6404·24	("β in phase of max. brightness") 330+24	354
14 5 50	6407·25	330+31, 410−50 "stars low"	360(?
Feb.			
8 14 0	6432·59	330−5 "v. low"	[325(?
17 14 15	6441·60	376−8, 330+20	363
19 14 10	6443·59	376+8 "v. low"	384(?
20 13 5	6444·55	348+12, 376−12	362
14 0	6444·59	330+12, 348−5	342
Mar.			
6 13 50	6458·58	330+20, 348+12, 376−12, 388−20	360
9 12 30	6461·53	388+16, 376+31, 410−16	402
24 13 15	6476·56	348−12, 330+8	337
14 10	6476·60	330−5 [δ Cygni+12]	(325(?
Apr.			
1 11 45	6484·50	348−12, 320+20, 330+12	339
5 11 0	6488·47	348+20, 376−8, "stars low, and strong)"	368(?
11 12 30	6494·53	348−5	343
17 14 0	6500·59	376+12, 388−5, 392−8, 410−20	386
14 15	6500·59	388−5	

* Half weight is given to these observations.

† "? is ξ Herc. variable?"

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1849 Apr. d h m			
26 11 35	6509·49	348−12, 320+20, 330+20	342
28 11 0	6511·47	388+5, 392−5	390
30 9 30	6513·40	388+12, 392+16, 410−16	401
13 5	6513·55	410−20, 392+12, 388−8	391
May			
4 9 50	6517·42	320+5, 348−20, 330+12 (" stars low ")	332(?)
13 15	6517·56	320+20, 348−12	338
14 35	6517·61	348+5, 330+24	354
5 13 10	6518·56	376+12, 388−12	382
6 9 55	6519·42	376+12, 388−8	383
7 10 50	6520·46	348−5	343
14 45	6520·62	320+16, 348−8, 330+20	342
8 10 35	6521·45	320+12, 348−12	334
12 30	6521·53	320+12, 348−12	334
1 12 15	6524·52	388+5, 376+12 392+8, 410−20	393
13 0	6524·55	410−16, 388+8, 376+20, 392+8	396
5 10 50	6528·46	330+20, 320+20, 348−8	343
13 10	6528·55	348−8	340
7 12 25	6530·52	348+20, 388−20, 376−5 " clouds "	369
3 10 15	6436·44	348+20, 376−12	366
13 50	6536·58	348+12, 376−12	362
5 11 40	6538·49	388+20, 410−20 " v. hazy "	399(?)
8 11 20	6541·48	320+12, 348−12, 330+8	335
June			
2 10 45	6546·46	348+5, 376−16	355
13 20	6546·56	348+8, 376−12	359

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1849 June d h m			
5 11 30	6549·49	376·5, 388−16	379
13 40	6549·57	348+20, 376−8	368
6 11 0	6550·47	388+24, 410−12	403
9 11 10	6553·48	330+12, 348−12, 320+12	337
13 0	6553·55	320+12, 348−16	332
10 12 0	6554·51	348+12, 376−16	360
11 11 40	6555·49	348−5, 376−20 (?+5)	[? 354] (350 ?)
12 12 0	6556·51	376+12, 388−5	384
13 11 45	6557·50	388+8, 376+20, 410−24	393
14 13 30	6558·57	348+20, 388−12, 376−8	371
16 10 45	6560·46	320+12, 348−12	334
17 11 30	6561·49	348+12, 376−12	362
13 15	6561·56	348+12, 376−12	362
18 11 15	6562·48	376+12, 388−12	382
19 12 45	6563·54	388+16, 410−16	399
13 50	6563·58	388+20, 410−5	406
23 10 55	6567·46	330−5	325
12 45	6567·54	330+8, 320−5 [*sic*]	(326(?))
24 11 30	6568·49	348+12, 376−8	365
28 11 0	6572·47	348−5, 330+20	344
30 13 20	6574·56	348−5, 330+16 " hazy "	344(?)
July			
1 12 30	6575·53	348+12, 376−8	365
3 12 50	6577·54	388+31, 410−8, 392+31	415
4 12 0	6578·51	348+20, 388−20, 376+5	372
8 11 15	6582·48	376+5	381
9 10 30	6583·45	388+12, 410−16	397
12 20	6583·52	410−20, 388+8, 392+8	395
10 10 35	6584·45	348+16, 376−12	364

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (Man. M.T.)	J.D. (Corrected.) 239 +	Corrected Comparisons.	Ded. Mag.
1849 July d h m			
13 12 20	6587·52	348+20, 376−12	366
14 12 30	6588·53	376+12, 388−8, 392−8	384
15 10 0	6589·43	410+12, 430−12	420
12 30	6589·53	410+8, 430−12	418
16 12 50	6590·54	388+20, 410−5	406
21 10 30	6595·45	376+12, 388−5, 392−5	386
23 10 10	6597·43	376+12, 388−12	382
27 10 15	6601·44	376+8, 388−12	381
30 12 35	6604·53	376+5	381
31 12 30	6605·53	348−12, 330+20	340
Aug.			
3 9 40	6608·41	388+8, 410−20	394
11 0	6608·47	388+5, 410−20	392
6 10 15	6611·44	320+20, 348−8, 330+16	342
11 10 45	6616·46	388+8, 410−12	397
12 10 5	6617·43	376+12, 388−8	383
16 9 45	6621·42	388+5, 392−12, 410−24	386
11 30	6621·49	388+8, 410−20	393
17 10 50	6622·46	388+5, 410−20	392
18 9 45	6623·42	376+20, 388−5	386
11 45	6623·50	376+5, 388−12	379
23 11 35	6628·49	410+12, 430−12	420
26 9 0	6631·38	348−12, 330+12	339
27 9 30	6632·41	348−12, 330+12	339
31 8 30	6636·36	348+12, 376−12	362
Sept.			
3 8 30	6639·36	348+20, 376−12	366
4 8 10	6640·35	376+12, 388−8	383
5 8 25	6641·36	410+8, 430−8	420
12 50	6641·54	410+5, 430−12	416
6 11 0	6642·47	388+8, 410−20	394
13 0	6642·55	376+12, 388−8	383
7 8 10	6643·35	376−5	371
10 50	6643·46	348+20, 376−12	366
8 11 30	6644·49	348−5, 330+20	344
11 10 0	6647·43	376+12, 388−5	383
14 8 5	6650·34	330+20 "clouds"	350
15 8 0	6651·34	348+12, 376−20	350
16 11 30	6652·49	376+5, 392−20	377
17 8 40	6653·36	388+12, 410−12	390
18 7 35	6654·33	388+12, 410−12	390
9 50	6654·42	388+20, 392+20, 410−5	400
12 20	6654·52	392+24, 410−8, 430−20	400
19 7 35	6655·33	376+20, 388−8	388
22 7 35	6658·32	348+20, 376−12	360
24 9 20	6660·40	376+20, 388−8, 410−24	388
25 8 15	6661·35	376+8, 388−12	388
26 10 20	6662·44	348+8, 376−12, 330+20	353
28 7 50	6664·33	348−8, 330+16	344
Oct.			
2 7 30	6668·32	376+12, 388−12	388
8 8 0	6674·33	376+16, 388−12	388
10 7 0	6676·30	348+12, 376−12	360
11 7 5	6677·30	348+20, 376−8	360
12 6 55	6678·29	348+20, 376−12	360
13 6 40	6679·28	388+20, 410−12	400
14 6 30	6680·28	388+31, 410−5	400
16 7 10	6682·30	348+20, 376−8	360
18 7 45	6684·33	348+20, 376−12	360
20 7 0	6686·30	388+8, 410−20	390
21 7 5	6687·30	388−5	383
28 7 35	6694·32	348+20, 376−5	371
Nov.			
2 6 0	6699·26	388+8, 410−20	390
3 6 5	6700·26	376+5, 388−12	383
6 6 30	6703·28	348+20, 376−12	360

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1849 Nov. d h m			
2 7 0	6709·30	376−5	371
5 6 35	6712·28	388+8, 410−31	392
6 6 0	6713·26	376+12, 388−8	383
3 10 20	6720·44	330+20, 410−50 "but low"	353
Dec.			
3 6 0	6730·26	376+5, 388−12	380
6 6 20	6733·27	348+31, 392−20, 330+½(410−330) or more	373(?)
4 9 20	6741·40	330+16 "but v. low"	346(?)
9 5 35	6746·24	376−8, 392−24	368
4 5 45	6751·25	392+8, 410−24	396
7 5 0	6754·22	330+12	342
8 5 40	6755·24	392−31, 330+31	361
1850 Apr.			
0 12 30	6858·53	348+20, 376−12	366
5 12 0	6873·51	410+12, 430−12	420
7 14 0	6875·59	348+5	353
May			
1 13 10	6879·55	376−5	371
9 11 45	6887·50	388+12, 410−16	397
2 12 0	6890·51	348+12, 376−20	358
4 12 30	6892·53	376+16, 388−12	383
0 12 15	6898·52	388−5	383
1 12 15	6899·52	410+8, 388+20	413
2 11 0	6900·47	388+..., 410−4	405(?)
6 13 30	6904·57	348+20, 376−12	366
9 12 25	6907·52	348+16, 376−16	362
1 12 10	6909·51	348+12, 376−20	358
June			
1 13 0	6910·55	348+16, 376−16	362
2 12 5	6911·51	376+20, 388−5, 410−20	390
3 12 5	6912·51	388−5 "(?)"	[383 ?]
4 11 45	6913·50	376+12, 388−8	383

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1850 June d h m			
18 10 35	6927·45	320+24, 348−12	339
22 11 15	6931·48	376+8, 388−5	384
26 10 15	6935·44	330+8	338
27 10 45	6936·46	348−5, 330+16	344
28 13 15	6937·56	376+12, 388−5	385
July			
4 10 50	6943·46	348+20, 376−8	368
5 12 10	6944·52	376+12, 388−8	383
7 11 5	6946·47	348+12, 376−8	365
8 11 45	6947·50	348−5, 330+16	344
9 11 0	6948·47	348−5, 330+16	344
12 15	6948·52	348−5, 330+12	342
13 11 15	6952·48	376+12, 388−5	384
14 11 15	6953·48	376+5, 388−12	379
15 11 45	6954·50	348+12, 376−12	362
20 10 30	6959·45	348+8, 376−12	359
21 11 5	6960·47	348−8, 330+16	342
22 10 55	6961·46	348−5, 330+20	344
11 45	6961·50	348−8, 330+16	342
24 10 0	6963·43	388+12, 410−12	399
28 9 45	6967·42	330+5	335
10 45	6967·46	348−12, 330−12	339
30 10 30	6969·45	376−5	371
Aug.			
3 11 55	6973·50	348−5	343
6 10 0	6976·43	388+16, 410−12	401
11 15	6976·48	388+12, 410−12	399
12 13 0	6982·55	376−8	368
14 10 25	6984·44	348+16, 376−12	364
16 11 15	6986·48	348+12, 376−20	358
17 11 45	6987·50	348−12, 330+12	339
18 10 35	6988·45	348+5, 376−20	354
19 9 20	6989·40	388+20, 410−8	404
10 15	6989·44	388+20, 410−5	406
20 10 0	6990·43	388+16, 410−12	401
10 40	6990·45	388+12, 410−12	399
21 9 45	6991·42	376+8, 388−20	379

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1850 Aug. d h m			
22 9 0	6992·38	348+12, 376−12	362
12 5	6992·51	348+12, 376−16	360
26 11 5	6996·47	388+5, 410−24	392
28 9 50	6998·42	348+20, 376−12	366
Sept.			
4 8 50	7005·37	348−12, 330+16	340
5 10 45	7006·46	330+12, 348−12	339
6 10 55	7007·46	348+12, 376−5	368
7 10 20	7008·44	376+8, 388−12	381
8 11 15	7009·48	388−5	383
9 9 35	7010·40	376+8, 388−12	381
10 9 0	7011·38	376+12, 388−12	382
11 9 0	7012·38	348+12, 376−12	362
12 8 5	7013·34	348+5, 376−20	354
10 25	7013·44	330+16, 348−5	344
13 8 0	7014·34	330+20, 348−5	344
14 9 20	7015·40	388+20, 410−8	404
12 30	7015·53	388+20, 410−12	402
15 7 20	7016·31	410+5	415
9 0	7016·38	388+12, 410−12	399
21 7 15	7022·31	376+8, 388−8	382
23 9 5	7024·38	330+8, 348−12	337
28 10 35	7029·45	388+12, 410−20	396
Oct.			
1 9 50	7032·42	330+16, 348−12	340
9 9 40	7040·41	348+20, 376−12	366
10 10 50	7041·46	388+20, 410−12	402
11 6 45	7042·29	388+16, 410−12	401
15 9 30	7046·40	348−8	340
21 7 10	7052·30	348+12, 376−20	358
23 7 10	7054·30	388+12, 410−12	399
10 15	7054·44	388+20, 410−12	402
28 6 35	7059·28	376−5	371
29 6 50	7060·29	388+5, 410−20	392
30 6 55	7061·29	388+12, 410−20	396
Nov.			
2 6 35	7064·28	348+12, 376−16	360
1850 Nov. d h m			
4 7 0	7066·30	388−5, 410−20	386
5 10 5	7067·43	388+..., 330+50, 410−16	391
23 5 35	7085·24	348+12, 376−8	365
25 5 55	7087·25	388−5	383
27 6 5	7089·26	348+12, 376−20	358
Dec.			
27 7 10	7119·31	330+50, 410−12	395
1851 Mar.			
6 15 45	7188·67	348+5, 330+20	352
14 12 50	7196·54	410+8, 430−12	418
20 13 15	7202·56	376+8, 388−12	381
Apr.			
7 10 25	7220·44	376−5	371
9 12 0	7222·51	388+20, 410−8	402
10 12 55	7223·54	376−5	371
11 13 25	7224·56	348+12, 376−12	362
15 12 10	7228·51	376+5	381
May			
3 12 40	7246·53	348+16, 376−12	36
13 12 30	7256·53	330+12, 348−12	339
19 10 25	7262·44	376−12, 348+12	362
27 10 40	7270·45	348+20, 376−12	366
28 11 5	7271·47	348+12, 376−12	362
29 10 45	7272·46	376−5	37
30 12 45	7273·54	388+20, 410−8	402
31 11 0	7274·47	388+12, 410−8	40
12 0	7274·51	388+12, 410−12	39
June			
1 10 30	7275·45	376+5, 388−20	37
14 12 5	7288·51	348+12, 376−16	36
15 11 5	7289·47	330+16, 348−12	34
17 11 0	7291·47	330+20, 348−8	34
25 13 30	7299·57	388+16, 410−12	40
27 11 15	7301·48	348+8, 388−12	36
29 11 0	7303·47	330+20, 348−8	34
30 11 15	7304·48	348+12, 376−16	36

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1851 July d h m			
4 10 50	7308·46	348+12, 376−12	362
14 10 35	7318·45	388−5	383
17 11 10	7321·47	348+5	353
18 11 5	7322·47	330+20, 348−8	343
21 11 0	7325·47	388+½(410−388)	399
30 10 15	7334·44	330+16, 348−12	340
Aug.			
5 10 30	7340·45	348+20, 376−8	368
7 10 45	7342·46	348+16, 376−12	364
13 11 10	7348·47	348+12, 376−12	362
15 11 15	7350·48	348+16, 376−8	366
19 11 20	7354·48	348+12, 376−12	362
23 9 50	7358·42	376+½(388−376)	382
24 10 20	7359·44	376+½(388−376)	382
25 9 15	7360·39	376−5	371
26 9 0	7361·38	376−8	368
30 11 0	7365·47	376+12, 388−12	382
Sept.			
2 9 35	7368·40	348+20, 376−12	366
3 9 25	7369·40	376+5, 388−16	379
4 10 5	7370·43	376−5	371
9 9 15	7375·39	348+16, 376−12	364
10 10 50	7376·46	388+8, 410−20	394
12 8 10	7378·35	376+12, 388−12	382
14 9 30	7380·40	348+20, 376−12	366
15 10 5	7381·43	348+12, 376−12	362
21 8 10	7387·35	330+20, 348−8	344
26 8 55	7392·38	376+12, 388−12	382
Oct.			
1 8 20	7397·35	376+16, 388−8	384
7 6 48	7403·29	388+31, 410−5	407
9 50	7403·42	388+20, 410−8	402
Nov.			
2 6 35	7429·28	410+8, 430−8	420
17 6 35	7444·28	348+12, 376−12	362
21 5 40	7448·24	388−5	383

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1852 Apr. d h m			
24 11 5	7603·47	376+5	381
25 10 0	7604·43	376−12	364
May			
19 11 35	7628·49	376−5	371
July			
4 11 5	7674·47	388+½(410−388)	399
18 10 35	7688·45	348+20, 376−12	366
22 11 10	7692·47	376−8	368
23 11 10	7693·47	376+8, 388−12	381
26 10 5	7696·43	330+8	338
27 10 15	7697·44	330+12, 348−16	338
28 10 30	7698·45	330+16, 348−5	344
Aug.			
1 9 55	7702·42	330+20, 348−5	344
3 10 0	7704·42	348+16, 376−12	364
4 10 5	7705·43	376−8	368
5 10 5	7706·43	388+5, 410−24	392
6 10 10	7707·43	376−5	371
8 10 55	7709·46	348+12, 376−12	362
10 10 40	7711·45	376+8	384
13 10 35	7714·45	376+8, 388−12	381
15 10 20	7716·44	348+8, 376−20	356
21 9 10	7722·39	330+20, 348−8	343
30 10 15	7731·44	376+8, 388−12	381
31 9 35	7732·41	376+12, 388−8	383
Sept.			
1 10 5	7733·43	348+20, 376−12	366
2 9 35	7734·41	330+20, 348−8	343
3 9 40	7735·41	348+12, 376−12	362
5 10 55	7737·46	376+12, 388−8	383
8 9 50	7740·42	376+5	381
9 10 5	7741·43	348+16, 376−12	364
13 8 0	7745·34	376+12, 388−12	382
16 8 40	7748·37	348+20, 376−12	366
17 8 35	7749·36	348+20, 376−12	366
30 8 25	7762·36	348+20, 376−8	368

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1852 Oct. d h m			
1 9 0	7763·38	376+8, 388−12	381
1853 May			
2 11 10	7976·47	376−5	371
7 10 55	7981·46	330+20, 348+5	352
10 11 5	7984·47	388+12, 415−20	398
12 11 20	7986·48	330+16, 348−5	344
14 11 55	7988·50	348+12, 376−12	362
16 11 25	7990·48	376+8, 388−12	381
17 11 55	7991·50	348+16, 376−12	364
19 12 15	7993·52	348+8, 376−16	357
20 12 10	7994·51	348−5	343
21 12 5	7995·51	348+16, 376−12	364
22 10 50	7996·46	388+½(415−388)	402
23 11 25	7997·48	388+12, 415−24	397
27 10 45	8001·46	348+20, 376−12	366
31 11 45	8005·50	376+5[? 376−5]	(381(?))
June			
1 12 50	8006·54	348−5	343
4 13 0	8009·55	410+12, 430−12	419
6 12 5	8011·51	348+20, 376−12	366
13 11 20	8018·48	348+12, 376−20	358
15 10 50	8020·46	348+8, 376−20	356
16 11 0	8021·47	376−5	371
17 11 0	8022·47	410+8, 415−8	412
19 10 35	8024·45	376−5	371
22 11 0	8027·47	376−8	368
30 11 50	8035·50	388+20, 410−12	402
July			
7 11 0	8042·47	376+12, 388−12	382
10 10 55	8045·46	330+12, 348−8	340
12 11 0	8047·47	388+8, 410−31	392
17 10 30	8052·45	348−5	343
19 12 0	8054·51	388+8, 410−31	392
22 11 5	8057·47	348+8, 376−20	356
24 11 15	8059·48	376+8	384
28 10 35	8063·45	376−8	368

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1853 Aug. d h m			
2 10 30	8068·45	388+8, 410−31	392
3 10 55	8069·46	376+5, 388−20	378
8 10 50	8074·46	410−8	402
9 10 45	8075·46	348+20, 376−12	366
20 11 5	8086·47	388+20, 410−12	402
28 10 10	8094·43	376+12, 388−12	382
Sept.			
2 10 25	8099·44	388+20, 410−8	404
5 8 45	8102·37	348+12, 376−16	360
7 9 10	8104·39	348+20, 376−12	366
10 10 50	8107·46	376+12, 388−12	382
13 9 40	8110·41	348+12, 376−12	362
15 10 50	8112·46	410+8	418
16 8 15	8113·35	388+20, 410−8	404
11 0	8113·47	388+12, 410−16	397
18 10 20	8115·44	348+12, 376−12	362
23 9 25	8120·40	376+12, 388−12	382
Oct.			
2 7 55	8129·34	376+5	381
30 8 15	8157·35	376−5	371
Nov.			
8 6 55	8166·29	348+20, 376−12	366
1854 Apr.			
15 11 45	8324·50	348+8	356
11 12 35	8324·53	348+12, 376−12	362
17 12 0	8326·51	376+12, 388−12	382
20 11 25	8329·48	348+⅝(376−348)	367
23 11 20	8332·48	410+8, 430−12	418
May			
8 13 25	8347·56	348+12, 376−12	362
22 11 10	8361·47	348+12, 376−12	362
23 12 5	8362·51	348+12, 376−12	362
26 11 25	8365·48	376+5	381
27 12 25	8366·52	348+12, 376−12	362
June			
3 12 10	8373·51	348+12, 376−16	360
15 12 5	8385·51	348+20, 376−12	366

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
July			
h m			
1 5	8419·47	348−5	343
1 45	8423·50	388+12, 410−24	395
10 50	8427·46	348+16, 376−12	364
11 40	8428·49	376+12, 388−12	382
Aug.			
11 5	8446·47	348+12, 376−12	362
9 30	8459·40	376+5	381
Sept.			
11 30	8464·49	348+12, 376−12	362
9 30	8468·40	388+5	393
9 35	8469·40	376−8	368
11 35	8470·49	348+12, 376−12	362
10 40	8471·45	330+12, 348−12	339
8 50	8472·37	330+12, 348−12	339
10 20	8473·44	388−5	383
9 10	8475·39	376+12, 388−12	382
10 45	8479·46	376+5	381
8 40	8482·37	376−8	368
8 15	8483·35	348+8	356
8 10	8486·35	376+12, 388−8	383
9 15	8487·39	388+12, 410−16	397
10 35	8489·45	376−5	371
7 15	8491·31	376−8	368
Oct.			
7 35	8495·32	348+12, 376−12	362
8 25	8499·36	388+8, 410−31	392
7 40	8500·33	410−5	405
7 45	8507·33	376+12, 388−12	382
7 20	8512·31	388+12, 410−20	396
10 30	8512·44	388+more than ½ (410−388)	[<399]
6 50	8518·29	376+5	381
Nov.			
6 20	8526·27	388+20, 410−12	402
6 20	8528·27	348+20, 376−12	366
6 40	8531·28	376+12, 388−12	382
6 35	8552·28	410+o or − . . ., 388+ . . .	
1854 Dec.			
d h m			
9 6 30	8562·28	348+8	356
18 6 40	8571·28	388+o or more, 410−31	[389(?)]
1855 Mar.			
24 14 30	8667·61	388+ . . ., 410−. . .	
Apr.			
21 11 40	8695·49	330+½(348−330)	339
22 10 30	8696·45	330+20, 348−5	344
29 11 50	8703·50	330+8, 348−20	335
May			
1 10 50	8705·46	330+16, 348+5	352
2 10 45	8706·46	410+o "about," 388+. . . "hazy and strong ☽"	(410(?))
3 12 5	8707·51	376−5	371
4 11 0	8708·47	348+12, 376−12	362
6 11 0	8710·47	348+⅓(376−348)	357
7 11 0	8711·47	348+⅔(376−348)	367
11 11 5	8715·47	376−5	371
16 10 45	8720·46	410−20, 388+8	394
17 12 0	8721·51	348+⅔(376−348)	367
24 10 20	8728·44	330+⅔(348−330)	337
25 11 45	8729·50	348−5	343
26 11 30	8730·49	348+½(376−348)	362
27 11 15	8731·48	376+½(388−376)	382
28 10 20	8732·44	410+8	418
11 0	8732·47	410+8, 430−24	415
29 12 15	8733·52	388−5	383
June			
7 11 20	8742·48	330+5, 348−20	334
8 10 30	8743·45	330+12, 348−12	339
11 45	8743·50	330+⅔(348−330)	342
9 11 20	8744·48	388−5	383
10 11 0	8745·47	388+31, 410−8	406
11 11 30	8746·49	376+12, 388−12	382
15 11 30	8750·49	348+⅔(376−348)	367
20 10 30	8755·45	330+12, 348−12	339

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1855 June d h m			
21 10 35	8756·45	330+16, 348−8	342
23 11 0	8758·47	388+31, 410−8	406
26 10 30	8761·45	330+16, 348−12	340
27 10 25	8762·44	348−5	343
28 11 0	8763·47	348+12, 376−12	362
29 10 40	8764·45	388−8	380
July			
4 10 20	8769·44	348+⅔(376−348)	367
6 11 0	8771·47	388+31, 410−8	406
7 11 30	8772·49	376−5	371
8 11 0	8773·47	348+½(376−348)	362
9 10 15	8774·44	330+⅔(348−330)	342
20 10 5	8785·43	376−5	371
31 11 35	8796·49	388−5	383
Aug.			
1 10 0	8797·42	410−5	405
2 10 0	8798·42	376−5	371
4 11 0	8800·47	348+⅗(376−348)	365
5 10 30	8801·45	348+⅔(376−348)	367
12 10 30	8808·45	376−5	371
24 10 40	8820·45	348+¾(376−348)	369
25 9 45	8821·42	376−5	371
28 11 20	8824·48	348+⅔(376−348)	367
29 9 15	8825·39	348+½(376−348)	362
30 10 15	8826·44	348+½(376−348)	362
31 9 30	8827·40	348+⅔(376−348)	367
Sept.			
5 9 45	8832·42	376+5	381
6 9 5	8833·38	348+⅔(376−348)	367
11 10 25	8838·44	348+¼(376−348)	355
21 9 45	8848·42	388+31, 410−8	406
24 8 10	8851·35	376−5	371
10 5	8851·42	348+¾(376−348)	369
25 8 0	8852·34	348+⅔(376−348)	367
29 9 30	8856·40	376−8	368
30 9 5	8857·39	348+20, 376−12	366

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1855 Oct d h m			
2 7 45	8859·33	348+½(376−348)	36
3 11 45	8860·50	388+0 or more, 410−24	(388
5 8 10	8862·35	388+½(410−388)	39
7 7 25	8864·31	348+⅔(376−348)	36
8 35	8864·36	348+½(376−348)	36
8 7 5	8865·30	348+½(376−348)	36
12 8 20	8869·35	376−8	368
14 7 15	8871·31	348+⅓(376−348)	35
16 6 40	8873·28	330+50, 410−20	38
17 7 15	8874·31	388+20, 410−12	40
18 7 15	8875·31	388−5, 410−31	38
19 7 20	8876·31	376−5	37
21 6 40	8878·28	348+⅓(376−348)	35
22 6 25	8879·27	348+..., 376−...	
23 8 45	8880·37	376+⅓(388−376)	38
24 7 40	8881·32	376+½(388−376)	38
27 7 40	8884·32	348+⅓(376−348)	35
31 6 55	8888·29	388+8, 410−31	39
Nov.			
1 6 35	8889·28	376+½(388−376)	38
2 6 50	8890·29	376−5	37
3 6 20	8891·27	348+⅖(376−348)	36
15 6 15	8903·27	348+20, 376−12	36
16 6 5	8904·26	348+12, 376−12	36
20 6 15	8908·27	376+5	38
24 7 35	8912·32	388+5, 410−20	39
25 5 30	8913·24	410+5, 430−12	41
8 15	8913·35	410−5	40
26 7 10	8914·30	376+⅜(388−376)	38
27 5 20	8915·23	376−5	37
28 5 45	8916·25	348+⅔(376−348)	36
30 6 15	8918·27	348+¾(376−348)	36
Dec.			
2 5 35	8920·24	376+⅜(388−376)	38
5 6 0	8923·26	348+⅓(376−348)	35
6 7 10	8924·30	376−5	37

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1855 Dec. d h m			
5 5 20	8925·23	388+12, 410−20	396
5 6 20	8927·27	376+⅔(388−376)	384
5 5 40	8929·24	330+5	335
5 5 10	8937·22	348+..., 376−...	
5 5 25	8938·23	410−30, 388+12	396
5 6 55	8939·29	410−0 or more, "hazy and ☽"	407(?)
5 5 35	8942·24	348+⅓(376−348)	357
1856 Jan.			
18 18 50	8954·79	348−5	343
19 19 0	8960·80	330+8, 348−12	337
6 10	8961·26	330+8	338
19 19 5	8970·81	376+⅓(388−376)	380
Feb.			
16 16 25	8989·69	376−8	368
18 18 20	8996·77	376+⅓(388−376)	380
Mar.			
15 15 0	9022·63	376+½(388−376)	382
15 15 5	9031·63	348−0 or more	
13 13 20	9035·56	388−5	383
12 12 50	9036·54	376−5	371
15 15 45	9037·67	348+⅔(376−348)	367
12 12 50	9038·54	348+½(376−348)	362
11 11 50	9040·50	348+½(376−348)	362
Apr.			
15 15 5	9042·63	388+20, 410−12	402
15 15 35	9045·65	348+⅓(376−348)	357
12 12 0	9052·51	348+5	353
10 10 35	9056·45	388+5	393
11 11 35	9067·49	388+½(410−388)	399
10 10 30	9068·44	410+5	415
May			
11 11 10	9071·47	348+⅔(376−348)	367
10 10 50	9072·46	348+¾(376−348)	369
10 10 35	9079·45	348+¾(376−348)	369
1856 May d h m			
10 13 30	9080·57	388+20, 410−8	404
17 12 15	9087·52	388+5, 410−24	392
18 11 35	9088·49	376−5	371
19 11 50	9089·50	348+¾(376−348)	369
20 11 35	9090·49	348+½(376−348)	362
22 11 20	9092·48	376+5	381
26 11 30	9096·49	348+⅓(376−348)	357
29 12 20	9099·52	376+⅔(388−376)	384
30 11 20	9100·48	376+5	381
June			
3 11 35	9104·49	348+⅔(376−348)	367
4 12 5	9105·51	348+⅔(376−348)	367
5 10 30	9106·45	388+20, 410−12	404
11 35	9106·49	388+31, 410−5	407
6 11 0	9107·47	410+5 "v. hazy"	415(?)
9 11 25	9110·48	330+12, 348−5	343
10 11 5	9111·47	330+20 "clouds"	350
11 11 5	9112·47	376+⅔(388−376)	384
14 11 5	9115·47	348+5	353
15 11 10	9116·47	330+5, 348−12	335
20 11 45	9121·50	348+⅔(376−348)	367
25 10 50	9126·46	376+..., 388−...	
28 11 15	9129·48	330−5	325
30 12 5	9131·51	376+⅓(388−376)	380
July			
1 10 25	9132·44	410+⅓(430−410)	417
2 10 15	9133·44	410−8	402
11 15	9133·48	410−8	402
3 10 35	9134·45	376+5	381
7 10 55	9138·46	376+½(388−376)	382
8 11 20	9139·48	376+⅔(388−376)	384
9 12 0	9140·51	376−5	371
12 12 5	9143·51	348*−8	340
13 11 15	9144·48	376+⅓(388−376)†	380

* "Colour of β (now in its phase of max. brightness) *bright white.*"
† Colour of β *bright white.*

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1856 July d h m			
14 10 5	9145·43	410+12	422
16 10 30	9147·45	348+⅓(376−348)	357
21 10 0	9152·43	388+5	393
22 10 25	9153·44	376−5	371
24 11 15	9155·48	348+⅓(376−348)	357
25 10 20	9156·44	348+½(376−348)	362
28 11 5	9159·47	388+5	393
29 10 0	9160·43	376−5	371
30 11 30	9161·49	348+⅔(376−348)	367
31 11 15	9162·48	376+⅜(388−376)	384(?)
Aug.			
1 11 15	9163·48	376−8	368
2 12 5	9164·51	376+½(388−376)	382
3 10 10	9165·43	388−5	383
4 11 35	9166·49	376+½(388−376)	382
5 11 5	9167·47	348+⅔(376−348)	367
6 10 35	9168·45	348+½(376−348)	362
8 11 5	9170·47	388+8, 410−20	394
11 10 0	9173·43	376−8	368
19 10 35	9181·45	348+⅔(376−348)	367
21 9 10	9183·39	388+⅖(410−388)	397
28 10 5	9190·43	388−8, 376+⅔(388−376)	382
29 10 5	9191·43	388+5	393
31 9 45	9193·42	348+⅔(376−348)	367
Sept.			
2 8 10	9195·35	348+⅓(376−348)	357
11 0	9195·47	348+⅓(376−348)	357
3 11 15	9196·48	388+⅓(410−388)	395
4 9 10	9197·38	410+5	415
11 5	9197·47	410−5	405
12 20	9197·52	410−8, 388+$\frac{9}{11}$(410−388)	404
5 9 5	9198·38	376+⅓(388−376)	380
6 11 0	9199·47	348+⅓(376−348)	357
7 8 35	9200·36	348+⅔(376−348)	367

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1856 Sept. d h m			
13 8 50	9206·37	348+½(376−348)	36
17 9 35	9210·41	388+..., 410−...	
18 7 50	9211·33	376+5	38
19 11 0	9212·47	348+⅔(376−348)	36
20 8 15	9213·35	348+½(376−348)	36
21 10 15	9214·44	348+½(376−348)	36
22 11 30	9215·49	376−8	36
24 10 14	9217·44	376−5	37
29 8 15	9222·35	388+¾(410−388)	40
9 40	9222·41	410−8	40
Oct.			
1 8 30	9224·36	348+¾(376−348)	39
4 7 30	9227·32	348+..., 376−...	
5 8 30	9228·36	376−8	39
12 8 15	9235·35	410−8	41
13 7 15	9236·31	410−8	41
17 6 35	9240·28	348+¾(376−348)	39
19 7 15	9242·31	388−5	39
10 35	9242·45	388−5	39
20 6 10	9243·26	348+$\frac{5}{7}$(376−348) "twilight and clouds"	39
22 7 30	9245·32	348+½(376−348)	39
24 10 45	9247·46	(fig.) 330+26; (sym.) 330+24	39
25 6 20	9248·27	388+½(410−388)	39
27 7 20	9250·31	348+⅔(376−348)	39
31 9 35	9254·41	(fig.) 330+26 "about"	39
Nov.			
6 6 40	9260·28	376+⅓(388−376)	39
7 7 20	9261·31	388+..., 410−12 "foggy"	39
8 8 30	9262·36	388+½(410−388), 410−20	39
9 7 20	9263·32	348+⅔(376−348)	39
10 6 20	9264·27	330+12, 348−8	39
11 6 30	9265·28	330+16, 348−5	39

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1856 Nov.			
d h m			
7 15	9266·31	330+12, 348−8	341
6 55	9267·29	330+31, 376+0 or more	(369(?))
5 50	9268·25	388+¼(410−388)	394
8 30	9268·36	388−5	383
7 10	9269·31	348+¾(376−348)	369
5 50	9278·25	348+½(376−348)	362
5 30	9281·24	388+8	396
5 45	9282·25	376−5	371
6 10	9283·26	348+⅓(376−348)	362
5 20	9284·23	330+12, 348+¼ (376−348)	349
Dec.			
6 0	9285·26	348+⅔(376−348), 330+12	360
5 40	9287·24	388+½(410−388)	399
6 40	9287·28	388+⅔(410−388)	397
7 40	9293·33	(fig.) 330+35	365
5 10	9294·22	(fig.) 330+39 "clouds and v. hazy"	369(?)
8 40	9298·37	(fig.) 330+18	348
5 40	9302·24	(fig.) 330+13; (sym.) 330+16	344
5 50	9302·25	348+..., 376−...	
6 20	9310·27	330+8	338
18 45	9310·79	330+12, 348−12	339
6 20	9312·27	(fig.) 330+35	365
(Comparisons henceforward are in figures, except when otherwise stated.)			
1857 Jan.			
6 20	9327·27	330+31 "hazy"	361(?)
6 50	9328·29	330+13	343
19 0	9328·80	348−5 (sym.)	343
6 40	9334·28	(With finder) 330+26 "about"; "β too low"	356(?)
18 45	9339·79	388−5 (sym.)	383
1857 Feb.			
d h m			
3 17 25	9349·73	330+5, 348−5	339
11 18 0	9357·76	388+5 (sym.), 410−20 (sym.)	392
14 17 45	9360·75	330+9, 348−5 (sym.)	342
20 16 5	9366·68	330+12 (sym.) "clouds interrupted"	342(?)
Mar.			
4 13 10	9378·55	410−5 (sym.), 388+... "stars low, but sky v. clear"	405(?)
13 40	9378·57	410−5 (sym.)	405
8 14 35	9382·61	330+70, 410−18 "about"	(394(?))
15 12 55	9389·54	330+31, 376+0 or−...	(368(?))
Apr.			
13 12 35	9418·53	376−8 (sym.)	368
14 12 0	9419·51	348+½(376−348)	362
15 11 45	9420·50	348−5 (sym.)	343
16 12 15	9421·52	348+⅔(376−348)	367
18 12 0	9423·51	376+½(388−376)	382
19 12 30	9424·53	376−8 (sym.)	368
20 10 50	9425·46	348+½(376−348)	362
23 11 25	9428·48	376−5 (sym.)	371
25 11 5	9430·47	388−5 (sym.)	383
26 11 55	9431·50	348+⅔(376−348)	367
27 12 35	9432·53	348+⅓(376−348)	357
May			
4 12 0	9439·51	330+18, 348−5 (sym.)	344
5 11 0	9440·47	348+½(376−348)	362
12 35	9440·53	348+⅔(376−348)	365
12 50	9440·54	330+28	358
6 11 30	9441·49	330+43, 410−26	380
7 9 50	9442·42	410+5 (sym.)	415
11 0	9442·47	410−2	408
15 11 45	9450·50	348+⅔(376−348)	367

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1857 May d h m			
18 11 30	9453·49	330+22 "clouds"	352
19 11 45	9454·50	376−5 (sym.)	371
21 12 40	9456·53	388−5	383
24 10 35	9459·45	348+5, 330+11	349(?)
26 12 15	9461·52	388−5 (sym.)	383
27 13 0	9462·55	330+39, 410−31	375
28 11 20	9463·48	376−9	367
30 11 5	9465·47	330+13, 348+2	349
12 45	9465·54	348−5 (sym.)	343
31 11 20	9466·48	376−5 (sym.)	371
June			
4 12 25	9470·52	330+⅔(348−330)	342
7 11 5	9473·47	376−5	371
12 11 50	9478·50	348+⅓(376−348)	357
13 13 5	9479·55	410−31	379
14 12 0	9480·51	388+22, 410−5	406
15 10 30	9481·45	410+5	415
12 0	9481·51	410+5 (sym.)	415
17 11 50	9483·50	376−9	367
18 12 0	9484·51	348+⅔(376−348), 376−11	365
19 11 5	9485·47	376−9	367
20 12 30	9486·53	348+½(376−348)	362
21 10 35	9487·45	376+½(388−376)	382
22 11 20	9488·48	376−9	367
24 11 45	9490·50	330+⅔(348−330)	342
25 11 20	9491·48	348−5 (sym.)	343
26 11 30	9492·49	348+⅔(376−348)	367
27 10 35	9493·45	410−9	401
28 10 0	9494·43	410+2	412
July			
1 12 45	9497·54	330+⅓(348−330)	336
10 11 45	9506·50	388+22, 410−5	406
11 11 15	9507·48	388+⅔(410−388)	403
12 11 30	9508·49	348−5 (sym.)	343
13 12 5	9509·51	348+⅓(376−348), 330+13	352
16 11 25	9512·48	376−5	371
19 11 15	9515·48	348+¾(376−348)	369
20 10 0	9516·43	330−5 (sym.)	325
11 15	9516·48	330−5 (sym.)	325
25 11 45	9521·50	376+½(388−376)	382
28 11 20	9524·48	348+⅜(376−348)	367
31 11 0	9527·47	330+22	352
Aug.			
2 11 45	9529·50	330+22, 348+½(376−348)	357
3 10 5	9530·43	348+⅜(376−348), 330+28	364
9 10 5	9536·43	330+13	343
13 11 15	9540·48	376+0	376
16 10 30	9543·45	376−5	371
18 11 5	9545·47	388+22, 410−7	405
20 10 40	9547·45	376−5 (sym.)	371
21 9 40	9548·41	376+0	376
22 10 45	9549·46	348+½(376−348)	362
23 10 15	9550·44	376+2	378
24 10 25	9551·44	388+0, 410−26	386
25 10 0	9552·43	388+0, 410−26	386
26 10 15	9553·44	376+0	376
27 10 25	9554·44	376−2	374
28 10 15	9555·44	348+½(376−348)	362
Sept.			
2 9 5	9560·38	376−2	374
3 10 40	9561·45	330+18	348
6 10 15	9564·44	330+52, 410−26 "clouds"	38
7 10 35	9565·45	388+0	388
9 11 0	9567·47	330+22	352
10 9 35	9568·41	376+0	376
12 8 15	9570·35	376+½(388−376)	382
13 10 5	9571·43	410+0 ("about") "hazy"	410
14 7 50	9572·33	410−13	397

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
57 Sept. d h m			
8 20	9574·35	330+5	335
8 10	9575·35	330+7	337
8 30	9576·36	376−5	371
7 55	9577·33	376−2	374
8 45	9578·37	388+5, 410−18	393
9 20	9580·39	348+½(376−348)	362
8 0	9581·34	348+½(376−348)	362
8 0	9582·34	330+35	365
9 30	9583·41	330+52("about"), 376+½(388−376)	382
8 40	9586·37	376+0	376
8 15	9587·35	330+22	352
Oct.			
8 25	9592·36	330+26, 348+⅔(376−348)	362
8 0	9594·34	330+9, 348+0	344
8 45	9596·37	388+0	388
11 0	9696·47	410−26	384
8 0	9602·34	330−18 "Query"	[312?]
6 35	9605·28	330+9	339(?)
10 45	9611·46	330+20	350(?)
6 35	9612·28	330+31	361
7 35	9614·32	330+35	365
7 45	9615·33	388+5	393
6 35	9616·28	388+0, 410−26	386
6 35	9618·28	348+⅔(376−348)	367
10 30	9619·45	330+13("about") "clouds"	343(?)
Nov.			
6 50	9620·29	330+18	348
7 10	9625·31	330+5	335
7 40	9630·33	330+36, 410−36	370
6 35	9634·28	348+⅔(376−348)	367
6 35	9637·28	376+½(388−376)	382
7 5	9642·30	410−26	384
6 25	9644·27	410−26	384
6 0	9645·26	330+13	343
1857 Nov. d h m			
27 5 40	9646·24	330+9	339
29 6 15	9648·27	410−7	403
Dec.			
4 7 10	9653·30	330+36, 410−36	370
7 5 15	9656·23	388+0	388
8 6 20	9657·27	330+36	366
13 5 30	9662·24	410−5 "about," "v. hazy"	405(?)
19 6 5	9668·26	330+44, 410−26	380
20 6 5	9669·26	410−22	388
25 7 0	9674·30	410+0	410
1858 Jan.			
4 6 10	9684·26	330+13	343
9 6 10	9689·26	330+26, 410−44	360
11 19 0	9691·80	330+9 "twilight"	339(?)
20 5 35	9700·24	410+0	410
21 6 0	9701·26	330+58 "sky rather thick"	[388(?)]
30 18 50	9710·79	330+31 "twilight too strong"	[361(?)]
Mar.			
6 14 45	9745·62	388+9, 410−13	397
7 13 45	9746·58	330+35, 410−35	370
16 13 50	9755·58	330+9	339
26 13 0	9765·55	330+80, 410+0 "moonlight"	410
Apr.			
1 13 15	9771·56	410−20, 388+0	389
4 12 30	9774·53	330+9, 348+0	344
9 14 30	9779·61	376−5	371
10 12 5	9780·51	330+18, 348+½(376−348)	355
12 13 20	9782·56	330+9	339
18 13 15	9788·56	330+18, 410−52, 348+0	351
20 11 0	9790·47	410+2	412
11 45	9790·50	410+0	410
21 12 45	9791·54	388+18, 410−9	403

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1858 Apr. d h m			
22 12 0	9792·51	330+18	348
23 13 15	9793·56	348+5	353
25 13 15	9795·56	330+11, 348+0	345
May			
1 12 45	9801·54	330+22, 348+⅓ (376−348)	355
2 11 30	9802·49	348+⅓ (376−348), 330+18	353
3 11 0	9803·47	388+⅓ (420−388)	395
4 11 0	9804·47	388+½ (420−388)	399
5 14 15	9805·60	376·5	371
6 11 30	9806·49	330+18, 348+0	348
10 12 30	9810·53	376+½ (388−376)	382
11 11 10	9811·47	376−9	367
12 10 40	9812·45	376+0	376
16 11 45	9816·50	388+0	388
18 12 15	9818·52	376+0	376
19 12 10	9819·51	348+0	348
21 11 40	9821·49	330+9, 348−5	342
22 11 40	9822·49	376+0, 410−26	380
31 11 15	9831·48	376−5	371
June			
1 12 45	9832·54	348+⅓ (376−348)	357
4 12 50	9835·54	348+½ (376−348)	362
5 11 45	9836·50	376+0 "γ Lyræ is evidently slightly variable"	376
6 11 20	9837·48	330+18, 348−5	344
7 12 25	9838·52	348−5	343
8 12 45	9839·54	330+18, 348−5	344
12 11 50	9843·50	388−9	379
13 11 15	9844·48	330+26, 348+9	356
14 11 45	9845·50	376+0 ("about"), 330+39	[372 ?]
17 12 0	9848·51	348+⅓ (376−348)	357
18 10 50	9849·46	388+0	388
20 11 55	9851·50	330+22, 348−5	345

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1858 June d h m			
21 12 0	9852·51	348+0	348
24 11 55	9855·50	388+11, 410−18	397
26 12 20	9857·52	330+18, 348+0	348
27 11 40	9858·49	376−5	371
July			
1 12 15	9862·52	388+0, 410−26	386
2 13 15	9863·56	376−7	369
6 11 50	9867·50	388+5, 410−22	390
10 11 45	9871·50	376+9	367
11 10 50	9872·46	348+0 ("about")	348
17 12 15	9878·52	348+5	353
18 12 20	9879·52	376−9	367
19 12 10	9880·51	376+0, 410−35	376
21 10 45	9882·46	388+0	388
29 13 0	9890·55	330+18	348
31 13 15	9892·56	330+26	356
Aug.			
5 11 0	9897·47	330+26, 348+⅓ (376−348)	357
6 10 15	9898·44	330+35, 348+½ (376−348)	364
7 10 30	9899·45	376+0	376
8 9 55	9900·42	388+0	388
9 12 35	9901·53	376+0 ("about")	376
10 11 15	9902·48	376+0	376
12 10 55	9904·46	348+⅔ (376−348)	367
21 10 45	9913·46	376+½ (388−376)	382
27 8 40	9919·37	410−5 "clouds"	405
12 0	9919·51	388+18, 410−9	403
28 10 45	9920·46	388+24, 410−5	406
Sept.			
4 10 0	9927·43	388+0	388
5 10 15	9928·44	376+½ (388−376)	382
11 11 15	9934·48	376+½ (388−376)	382
12 11 20	9935·48	376+0	376
13 10 15	9936·44	348+⅔ (376−348)	367
16 10 0	9939·43	388+0	388

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (Man. M.T.)	J.D. (Corrected.) 239+	Corrected Comparisons.	Ded. Mag.
1858 Sept.			
d h m			
9 15	9940·39	376+½(388−376)	382
9 40	9943·41	330+5	335
10 20	9944·44	376+0	376
12 15	9945·52	388+…, 410−13	397
9 0	9947·38	376+0	376
8 5	9948·34	376−9	367
11 40	9948·49	330+5	335
9 25	9949·40	330+7	337
10 20	9953·44	376+0	376
Oct.			
4 8 5	9957·34	330+13	343
5 9 30	9958·40	410−9	401
1 8 50	9964·37	388+0	388
3 7 55	9966·33	388+8, 410−20	395
7 9 45	9970·41	410−31	379
0 9 30	9973·40	330+22	352
3 7 0	9976·30	330+13	343
8 7 35	9981·32	376−9	367
Nov.			
3 6 45	9987·29	330+5	335
5 8 45	9989·37	388+0	388
6 7 15	9990·31	388+0	388
7 7 30	9991·32	388+5, 410−22	390
8 9 20	9992·40	330+22	352
11 7 0	9995·30	376+0	376
13 7 5	9997·30	388+9, 410−22	394
14 6 0	9998·26	388+18, 410−13 "clouds"	401
15 9 30	9999·40	330+35	365
	240+		
17 6 45	0001·29	330+18	348
21 6 15	0005·27	330+31	361
22 9 0	0006·38	330+44, 410−26	380
23 6 0	0007·26	330+26, 410−44	360
24 7 15	0008·31	330+18	348
26 6 15	0010·27	388+½(410−388)	399
28 7 0	0012·30	330+13	343

Date. (Man. M.T.)	J.D. 240+	Corrected Comparisons.	Ded. Mag.
1858 Dec.			
d h m			
2 6 10	0016·26	376+0	376
5 6 20	0019·27	376+0	376
22 6 20	0036·27	388+5, 410−26	392
24 7 20	0038·31	330+5 ?	335(?)
29 7 0	0043·30	330+22	352(?)
G.M.T.			
1859 Jan.			
7 6 20	0052·26	330+5	335
Feb.			
28 15 5	0104·63	348+0	348
Mar.			
31 12 55	0135·54	330+5	335
Apr.			
4 12 55	0139·54	388+18, 410−5	405
6 14 5	0141·59	348+⅔(376−348)	367
9 11 30	0144·48	348+0	348
17 10 35	0152·44	410+2	412
18 11 15	0153·47	388+9, 410−13	397
21 11 0	0156·46	348+5	353
22 10 30	0157·44	348+0 ("about")	(348(?))
26 10 15	0161·43	348+0	348
May			
1 11 15	0166·47	348+⅔(376−348)	367
2 13 30	0167·56	348+½(376−348)	362
4 12 0	0169·50	330+5	335
8 13 30	0173·56	330+5	335
11 12 40	0176·53	330+13	343
12 11 30	0177·48	388+0, 410−26	386
16 12 0	0181·50	330+5	335
17 10 45	0182·45	348+5	353
19 10 15	0184·43	388+13, 410−13	399
20 10 45	0185·45	376+0	376
21 12 15	0186·51	376−5	371
23 13 5	0188·54	348+⅔(376−348)	367
25 13 0	0190·54	410−5	405
27 13 0	0192·54	388+0	388

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (G.M.T.)	J.D. 240+	Corrected Comparisons.	Ded. Mag.
1859 May d h m			
28 13 30	0193·56	348+⅓(376−348)	357
29 12 35	0194·52	376−9	367
31 12 0	0196·50	376+0	376
June			
7 10 45	0203·45	410−9	401
8 12 15	0204·51	388+22, 410−9	404
9 10 50	0205·45	330+26	356
10 12 10	0206·51	348+0	348
14 12 50	0210·53	376−9	367
15 12 40	0211·53	376−9	367
16 13 0	0212·54	330+13	343
17 13 35	0213·56	330+13	343
20 12 15	0216·51	410−5	405
22 11 45	0218·49	376−9	367
29 12 50	0225·53	376+0	376
30 12 10	0226·51	348+0	348
July			
9 11 25	0235·48	376+0	376
10 11 45	0236·49	410−18	392
11 13 35	0237·56	376−9	367
12 13 50	0238·57	348+½(376−348)	362
13 13 35	0239·56	330+9	339
14 11 10	0240·47	330+9	339
15 11 50	0241·48	388+0	388
17 10 50	0243·45	410−9	401
19 11 55	0245·50	348+7	355
20 11 30	0246·48	330+5	335
24 14 50	0250·62	330+39	369
29 14 10	0255·59	410−5	405
Aug.			
1 11 45	0258·49	330+5	335
2 11 35	0259·48	376−5	371
10 12 55	0267·54	376+0	376
14 11 0	0271·46	330+26	356
18 12 15	0275·51	376+½(388−376)	382
19 11 20	0276·47	388+. . .	<388
21 10 30	0278·44	376−5	371
1859 Aug. d h m			
23 9 10	0280·38	376+0	376
24 13 0	0281·54	388+18, 410−9	403
Sept.			
3 9 0	0291·38	376+½(388−376)	382
10 10 50	0298·45	330+13	343
14 11 50	0302·49	330+26	356
16 10 10	0304·42	330+13	343
17 10 5	0305·42	330+9	339
21 9 10	0309·38	330+13	343
22 10 30	0310·44	376−9	367
26 11 30	0314·48	330+26	356
28 8 35	0316·36	376+0	376
29 10 5	0317·42	330+22	352
Oct.			
3 9 35	0321·40	376+0	376
21 9 30	0339·40	388+5	393
26 10 5	0344·42	330+22	352
27 9 45	0345·41	330+66, 410−22	390
29 10 45	0347·45	330+39	369
Nov.			
7 6 30	0356·27	330+9	339
8 6 15	0357·26	330+26	356
9 8 45	0358·36	410−5	405
16 6 35	0365·27	388+9, 410−22	394
20 6 40	0369·28	330+13	343
24 8 35	0373·35	330+31	361
30 ..	0379–80	376−9	367
Dec.			
1 ..	0380–81	330+5, 376+0	356(
5 ..	0384–85	410−9	401
14 ..	0393–94	330+18	348
1860 Feb.			
6 14 35	0447·61	330+36, 410−39 "v. low"	368
Mar.			
14 13 15	0484·55	348+½(376−348)	362
Apr.			
5 12 30	0506·52	410−18	392

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (G.M.T.)	J.D. 240+	Corrected Comparisons.	Ded. Mag.
1860 Apr.			
d h m			
12 15	0509·51	348+0	348
12 35	0512·52	388+0	388
11 40	0517·49	348+½(376−348)	362
12 20	0518·51	388+0	388
11 5	0524·46	348+0	348
May			
11 5	0532·46	376+0	376
12 20	0534·51	348+½(376−348)	362
12 25	0537·50	348+⅓(376−348)	357
12 55	0540·54	376+0	376
11 20	0544·47	376−9	367
11 30	0547·48	348+9	357
13 40	0548·57	376−9	367
June			
11 5	0568·46	330+13	343
12 0	0572·50	388+0	388
12 30	0580·52	330+13	343
12 25	0589·52	376+0	376
July			
11 55	0593·50	330+13	343
11 55	0594·50	330+9	339
11 35	0596·48	376+0	376
13 15	0600·55	330+9	339
11 30	0614·48	348+0	348
11 45	0616·49	388+22, 410−5	406
11 40	0620·49	330+13	343
14 10	0621·59	348+0	348
10 40	0622·45	348+5	353
Aug.			
12 55	0629·54	388+18, 410−13	401
Sept.			
12 0	0664·50	348+0	348
10 0	0667·42	388−5	383
9 0	0679·38	348+9	357
Oct.			
9 15	0692·39	348+0	348
9 35	0695·40	388+0	388
1860 Oct.			
d h m			
17 8 15	0701·34	376+0	376
20 7 45	0704·32	348+9	357
21 10 30	0705·44	330+9	339
23 9 45	0707·41	410−9	401
Nov.			
1 8 20	0716·35	330+18	348
2 9 45	0717·41	330+9	339
3 7 5	0718·29	376−5	371
5 7 10	0720·30	388+9, 410−22	394
17 7 40	0732·32	376+0	376
1861 Mar.			
19 13 20	0854·55	348+½(376−348)	362
31 12 15	0866·51	330+22	352
Apr.			
6 14 15	0872·59	348+0	348
8 12 30	0874·52	388+9, 410−18	395
9 11 45	0875·49	388+18, 410−18	399
16 11 45	0882·49	376−9	367
17 12 5	0883·50	348+9	357
29 13 15	0895·55	376−5	371
May			
4 13 30	0900·56	388+0	388
9 13 30	0905·56	376−9	367
14 11 15	0910·47	348+0	348
17 13 0	0913·54	388+13, 410−13	399
27 11 30	0923·48	376−5	371
29 11 30	0925·48	376−13	363
30 12 0	0926·50	388+18, 410−9	403
June			
14 12 30	0941·52	348+5	353
15 13 5	0942·54	348−5	343
16 13 40	0943·57	348+5	353
17 11 40	0944·49	348+5	353
19 11 40	0946·49	388+9, 410−18	395
25 12 10	0952·51	410−5	405
26 11 30	0953·48	388+13, 410−13	399

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (G.M.T.)	J.D. 240+	Corrected Comparisons.	Ded. Mag.
1861 July			
d h m			
8 12 10	0965·51	410−5	405
Aug.			
5 10 0	0993·42	330+0	330
6 11 35	0994·48	330+13, 348+0	346
21 10 10	1009·43	348+0	348
Sept.			
1 9 45	1020·41	376−5	371
20 9 20	1039·39	330+18	348
Oct.			
11 7 20	1060·31	376−9	367
16 9 20	1065·39	330+35	365
Nov.			
15 7 30	1095·31	410−5	405
17 8 15	1097·34	330+35	365
Dec.			
3 8 25	1113·35	330+60, 410−44	376
1862 Apr.			
23 11 0	1254·46	348+0	348
29 12 25	1260·52	348+9, 376−18	357
June			
16 11 40	1308·49	376+0	376
17 13 20	1309·55	376−5	371
July			
4 12 15	1326·51	388+18, 410−9 "v. hazy"	403(?)
12 11 20	1334·47	376+0	376
19 11 0	1341·46	388+0	388
24 13 10	1346·55	388+5, 410−26	392
28 12 45	1350·53	388+0	388(?)
Sept.			
21 9 30	1405·40	388+18, 410−13	401
30 10 35	1414·44	330+26	356
Oct.			
12 ..	1426–27	330+18	(348)
14 (before 10 45)	1428–29	330+26, 348+9	(357)
15 (before 10)	1429–30	330+39	(369)
29 6 50	1443·28	388+26, 410−9	404
1862 Nov.			
d h m			
15 8 0	1460·33	330+5	335
16 8 0	1461·33	330+9	339
1863 Feb.			
14 14 55	1551·62	376−9	367
Apr.			
22 13 55	1618·58	376−9	367
29 11 25	1625·48	376+0	376
May			
8 12 35	1634·52	330+26, 348+0	352
23 13 15	1649·55	388+26, 410−5	406
24 10 40	1650·45	410+0	410
25 14 0	1651·58	376+0, 410−35	376
1865 May			
15 10 37	2372·44	410+0	410
June			
10 10 48	2398·45	410−2	408
11 10 35	2399·44	410+0	410
11 25	2399·48	410−9	401
17 ..	2405–06	410−31, 388+0	(384
23 12 15	2411·51	388+18, 410−18	399
1866 Apr.			
29 10 45	2721·45	410+0 "o. and hazy sky"	41
30 10 40	2722·45	410+18 [? 410−18]	(42
May			
23 12 30	2745·52	348+9, 330+26	35
26 12 20	2748·59	376+9	36
July			
16 11 35	2799·48	410+0	41
22 13 35	2805·56	388+0	38
Aug.			
3 11 15	2817·47	376−5	37
10 10 35	2824·44	388+22, 410−5	40
31 12 25	2845·52	410−5, 388+0	39
1867 May			
21 12 40	3108·53	410+0	41

TABLE IV.—*Baxendell's Observations of β Lyræ*—continued.

Date. (G.M.T.)	J.D. 240+	Corrected Comparisons.	Ded. Mag.
1857 June d h m			
12 0	3122·50	410−13	397
1858 May			
11 50	3469·49	410−18	392
1870 June			
11 10	4219·47	410−5 "Op. Gl."	405
Aug.			
10 20	4301·43	376−5 "Op. Gl."	371
11 0	4304·46	348+9	357
9 0	4305·38	348+9	357
10 25	4306·44	348−9	339
Sept.			
10 30	4319·44	348+9, 376−18	357
9 20	4320·39	348+0	348
10 20	4326·43	388−18	370
11 10	4333·47	330+18, 348−. . .	346(?)
Oct.			
12 10	4337·51	330+44	374
10 20	4338·43	330+18	348
1877 Sept. d h m			
3 9 45	6866·41	410+0	410
4 8 30	6867·35	410−36, 348+0 [? 376+0]	[? 361 or 375]
5 8 40	6868·36	348+9	357
7 10 10	6870·43	348+18, 376+0	371
8 10 0	6871·42	348+18, 376−9	367
9 9 15	6872·39	388+9, 410−36	392
12 8 40	6875·36	348+0	348
15 8 25	6878·35	388+18, 410−9	403
16 9 5	6879·38	410−5	405
17 8 0	6880·33	388+0, 410−36	381
20 9 10	6883·38	320+9, 348−9	334
21 8 0	6884·33	348+9, 388−9	368
23 8 20	6886·35	388+0	388
24 8 45	6887·37	348+0	348
Oct.			
8 8 0	6901·33	388+0	388

NOTE.—All the above observations except those in *square* brackets have been used in obtaining the mean curve, but some of those with queries have been given less weight. Those in *curved* brackets have, in addition, not been used in taking the residuals.

These observations extend over more than 1000 cycles (E = −421 to E = +643), but with many gaps. A period of 12·908 days was first assumed, but when the observations were tabulated accordingly it became clear that the period was continually increasing, and ultimately the following formula was deduced for the occurrence of the principal minimum :

$$2398590{\cdot}57 + 12{\cdot}908E + {\cdot}0000037E^2.$$

The initial date for each cycle was calculated from this formula to 0·01 day, and the cycle was first divided into ten equal portions of nearly 1·291 days each. But it was afterwards considered desirable to subdivide several of these into two, as shown in Table V. (Other of the portions were also experimentally subdivided, and portions 3 and 8 were also divided into three and into four parts, without affecting the general results.)

The series of observations is divided into two parts : E = −421 to −2, with 518 observations ; and E = +5 to +643, with 708 observations ; the cycles −1 to +4 were not observed.

The curve comes out sensibly symmetrical, and the figures are arranged to bring out this fact.

TABLE V.

Mean Light Curve of β Lyræ (*unit* 0·01 *mag.*).

Portion.	−421 to −2.	+5 to +643.	All.	Portion.	−421 to −2.	+5 to +643.	All.	Adopted for Mean Curve.
1 (*a*)	406	406	406	10 (*b*)	407	405	406	406
1 (*b*)	389	387	388	10 (*a*)	388	388	388	388
2	366	361	363	9	362	360	361	361
3 (*a*)	349	354	353	8 (*b*)	353	351	352	352
3 (*b*)	349	354	352	8 (*a*)	352	351	351	352
4	362	361	361	7	360	362	361	361
5	383	380	381	6	382	377	379	380

So far no other curve of β Lyræ had been referred to, in order that any possible bias might be avoided, but after this point reference was made to the *B.A.A.J.* (**33**, No. 7, 281–4), and it was gratifying to find that the curve there given, though not quite symmetrical, is in good general agreement with that above deduced.

[The B.A.A. formula is

$$2398590{\cdot}604 + 12{\cdot}908006\mathrm{E} + 0{\cdot}000003946\mathrm{E}^2,$$

and is probably to be preferred, although mine, given above, suits Baxendell's observations very well.]

Thus the *mean* curve of variation may be regarded as fairly well determined.

When it had been plotted, and 12 equidistant points read off, harmonic analysis gave the expression

$$7\cos\theta + 23\cos(2\theta) + 4\cos(3\theta) + 5\cos(4\theta).$$

[It is rather curious that the analysis of the "B" curve of R Sagittæ (in which the minima become equal) gives

$$23\cos(2\theta - 15^\circ) + 4\cos(4\theta - 74^\circ).]$$

(See *M.N.*, **76**, No. 8, 683.)

But this does not represent the whole of the facts. There are very large deviations from the mean, which are clearly for the most part not accidental but due to other systematic variations. To give one instance, the maximum (omitting a few very extreme cases which may be mistakes) ranges from 3·35 to 3·67, but is almost always within 0·10 of one of these extremes, and very rarely within 0·06 of the mid-value 3·51.

In order to investigate these deviations, each individual observation was compared with the mean curve, and all the residuals tabulated.

On inspection of these residuals two systematic deviations became evident.

(A) One of them appeared to be a seasonal effect of some kind, possibly due to the orientation of the field according to hour-angle. There seemed to be a systematic excess or defect in the recorded brightness for weeks or even months at a time.

The residuals were therefore collected according to calendar months

(omitting some scattered observations), and the mean result showed a definite and unmistakable systematic variation, certain months having always the negative and others always the positive sign. But the arrangement of the figures was irregular, and consideration showed that this was sure to be the case if the variation were due to the hour-angle, as the hour-angle at the times of observation varied by three or four hours during some months, and sometimes indeed during a single night, when several observations were taken. The residuals were therefore rearranged according to hour-angle, this time including the scattered ones, and Table VI. shows the results. The number of observations is given in brackets, and only where these exceed 100 can the mean be considered trustworthy.

TABLE VI.

Hour-Angle.	0 to +30°.	+30° to 60°.	+60° to 90°.	+90° to 120°.	+120° to 150°.	150° to 180°.	180° to −150°.	−150° to 120°.	−120° to 90°.	−90° to 60°.	−60° to 30°.	−30° to 0°.
:s.												
0–4	+1 (7)	+5(13)	+6 (2)	+12 (1)	...	...	...	...	+13(1)	+1 (8)	+0 (2)	+4 (1)
8–9	+4(51)	+6(52)	+4(32)	− 8 (8)	−6(1)	...	...	...	−4(11)	−11(9)	−6(16)	−1(22)
0–4	+1(89)	+3(70)	−4(12)	−17 (2)	...	...	...	...	+4 (3)	−2(18)	−2(38)	−3(42)
5–7	+4(80)	+3(83)	−1(34)	−15(12)	...	...	...	...	+3 (6)	−4(40)	−2(52)	−2(51)
8–61	+1(38)	−2(61)	−5(30)	− 7(11)	...	...	...	...	−2 (5)	−3(41)	−6(39)	−2(46)
2–77	+1(17)	+4(13)	−7 (6)	+17 (1)	...	...	...	...	+1 (1)	−2 (4)	0(11)	+1 (9)
an	+2	+3	−1	−9	[−6]	...	...	...	0	−3	−3	−2
of Obsns.	(282)	(292)	(116)	(35)	(1)]	...	...	...	(27)	(120)	(158)	(171)
m rmula }	+2	+3	0	−5	=6	...	...	0	−1	−4	−3	−2
C.	0	0	−1	−4	[0]	...	...	...	+1	+1	0	0

The mean figures strongly suggest a two-fold cause of variation: (1) changes in orientation; (2) changes in altitude. In going through the residuals, it was almost startling to find the very sudden change from the negative to the positive sign at the star's southing, and also the great and rather sudden apparent brightening (in most cases) at nearly six hours after southing. In many instances, in the columns next preceding and following the gap, the observer notes "stars v. low." The orientation effect appears to be reversed in the lower half of the star's course.

A purely empirical formula

$$3{\cdot}2 \cos (2h - 90^\circ) - 3{\cdot}5 \cos^4 (\text{alt.})$$

was deduced, and the last two lines in the table give its value and a comparison of it with the observed means. The observations were taken mainly in the neighbourhood of Manchester. The altitude has been taken only approximately.

This effect seems to be real, but for the most part small. The corrections indicated have not been applied to the observations or to the residuals. There may also be a small long-period variation, or slight changes in the observer's habit.

(B) A secondary variation of magnitude with a period near half that

of the main wave was easily perceived on careful investigation. Much time was spent in trying to identify this periodicity by the method of trial and error, without accomplishing much more than that it seemed to be between 6·46 and 6·66 days. The combined effects of the "seasonal" deviations and of the many gaps baffled all attempts at obtaining a long homogeneous series. Ultimately a number of short groups (from 4 to 14) of half-cycles were formed, as in Table VII. (starting always close to a primary or secondary min. of the main wave), wherever a sufficient number of observations with more or less the same seasonal error could be taken. The assumed period was 6·454 days—half the *mean* principal wave—and the interval between the groups always consists of an exact multiple of this. In Table VII. the first column (N) gives the number of these half-cycles elapsed from the mean of the first group to the mean of each in succession. The next six give the mean residual in each of six equal subdivisions of the 6·454 days; the next the number of observations in the group, and the next the position of its minimum, harmonically analysed in the usual way, the

TABLE VII.

	N.	(1).	(2).	(3).	(4).	(5).	(6).	No. of Obs.	Position of Min.	C_1.	O−C_1.	CF.	O−CF.
									°	°	°	°	°
1	0	0	+ 3	+ 6	+11	...	− 1	13	164	164	0	+45	−45
2	400	+4	+ 5	+10	+ 5	+ 5	+ 1	19	125	12	+113	+19	+94
3	436	−5	−16	−17	− 5	+ 3	+ 4	26	272	258	+ 14	+40	−26
4	445	+5	0	− 8	− 3	+ 5	+10	48	308	319	− 11	+45	−56
5	506	0	+ 7	+ 2	− 6	− 8	+ 1	28	47	15	+ 32	+57	−25
6	562	−1	+ 7	+17	+ 8	+ 2	−12	22	126	37	+ 89	+40	+49
7	666	+4	+ 2	− 3	− 8	− 3	+ 3	17	257	386	− 29	−33	+ 4
8	734	+2	+ 5	+ 9	+ 1	− 3	− 8	15	99	130	− 31	−72	+41
9	777	−3	− 5	−14	−12	− 8	− 2	25	335	423	− 88	−79	− 9
10	795	−2	+ 2	+11	+ 9	+ 3	− 3	16	149	186	− 37	−77	+40
11	834	+8	+ 3	−13	−14	− 6	+ 7	19	345	452	−107	−64	−43
12	845	+4	+ 9	+13	+ 9	+ 4	+ 3	23	120	167	− 47	−58	+11
13	857	−6	− 3	− 2	+ 5	− 3	− 4	28	172	249	− 77	−51	−26
14	884	−3	0	+ 5	− 3	− 8	− 9	14	99	73	+ 26	−32	+58
15	901	+2	+ 5	+ 8	+13	+ 8	+ 3	19	173	189	− 16	−19	+ 3
16	945	−6	− 3	− 1	− 5	−12	−14	27	99	129	− 30	+13	−43
17	970	0	− 4	− 1	+ 2	+15	+ 1	14	240	299	− 59	+31	−90
18	1002	−3	− 2	− 1	+10	+ 6	0	20	205	158	+ 47	+47	0
19	1056	−6	−13	− 4	+ 1	+ 8	+ 1	14	236	166	+ 70	+57	+13
20	1110	−6	−13	− 7	− 2	0	− 1	16	245	174	+ 71	+42	+29
21	1176	0	+ 8	...	+35	...	+ 7	7	178	264	− 86	− 2	−84
22	1637	−5	−13	− 6	0	+ 6	+ 4	7	249	168	+ 81	+52	+29
23	2035	−1	+ 3	+ 3	− 1	...	− 5	11	90	3	+ 87	+ 3	+84

gaps in Nos. 1, 21, and 23 being filled in by interpolation. (As each division contains 60°, there is considerable room for error in the analysis, especially where the observations are scanty.)

A calculation of the advances in phase between each of the groups, from No. 2 to No. 20, suggested a mean advance of 0·122 days (= 6°·8) per cycle of 6·454 days, giving a corrected period of 6·576 days. But when the first and last groups were added, a mean correction of 6°·82 was found to suit them better.

The next column (C_1) gives the phase obtained by adding 6°·82 N. (less any complete multiples of 360°) to 164°, the position of the minimum of No. 1. The following column gives $O - C_1$. Considering the meagreness of the data, the results are not bad, but there appears to be a systematic variation which might indicate a fluctuation or a gradual increase in the period. Eventually a fluctuation was tried, completed in $552 \times 6{\cdot}454$ days. CF gives the resulting correction of the minimum phase for each group, and the final column gives O – CF. The complete formula for correction of phase would thus be

$$+6^\circ{\cdot}82\mathrm{N} + 68^\circ \cos(\theta + 35^\circ) - 11^\circ,$$

where θ equals 360° N/552, or about 0°·652 N.

The groups Nos. 21 and 22 were formed merely in order to try to bridge the long gap between Nos. 20 and 23, and they cannot be considered trustworthy so far as precise details are concerned ; in fact, they can hardly be taken very seriously, only seven observations being available in each case. One observation in each of them is not improbably a mistake.

Several slightly differing forms of a regular increase of period were also tried, which gave residuals better than $O - C_1$ but not so good as O – CF. Fluctuations of rather shorter duration give nearly as good results as the one adopted above.

The accordance in the final column of Table VII. appears very satisfactory, considering the nature of the material, as about half the residuals are less than ±30, only four exceed ±60°, and only two exceed ±90°. These two are Nos. 2 and 17, and a glance at their details shows that their discordance is not very surprising. Nevertheless, the fluctuation formula must be considered as merely tentative. The really effective data extend only from N = 400 to N = 1110—less than $1\frac{1}{2}$ cycles of the assumed fluctuation,—and it is possible that this B wave is really sensibly constant, and the appearance of fluctuation merely accidental, or that it is regularly increasing like the main variation of β Lyræ, or that it fluctuates irregularly. There can, however, be no reasonable doubt of its real existence during these observations, nor that its mean length was somewhere near 6·576 days. Any definite conclusions on the other points must await the result of an investigation of other observations.

The β Lyræ curve in the *B.A.A.J.* (mentioned above) shows some curious short rises, which may quite possibly be due to this variation, especially as they occur at different points in the curve in the open-dot and black-dot series. I should greatly esteem an opportunity of examining the actual observations, with a view to testing this possibility and discovering whether this secondary variation still persists.

In order to gain some idea of the mean form and amplitude of this variation, the different examples were plotted on squared paper, and intermediate points read off between those already given, thus obtaining twelve equidistant points instead of six.

In Table VIII. these are arranged one below another, shifted (from their positions in Table VII.) in such a manner as to make their phases coincide as nearly as possible. It was necessary to have two divisions, as the phases in some of the examples do not fit in with those in some of the others. Each alternate figure is, of course, the same as in Table VII.

The first column in Table VIII. gives the number of the example (corresponding to that in the previous table); the next twelve the figures of the 12 equidistant points; the next the mean residual in each example; and the final column the year and months in which the observations were taken.

TABLE VIII.

Means of the Examples in Table VII.

No.	(1).	(2).	(3).	(4).	(5).	(6).	(7).	(8).	(9).	(10).	(11).	(12).	Mean Resid.	Date.
						a. *Examples with Max. and Min. nearly at a point.*								
2	+5	+ 5	+ 3	+ 1	+ 2	+ 4	+4	+5	+ 8	+10	+ 9	+5	+5	1848 Oct.
3	−5	−11	−16	−17	−17	−12	−5	+1	+ 3	+ 5	+ 4	+1	−6	1849 May–Ju
4	+2	0	− 5	− 8	− 7	− 3	+2	+5	+ 9	+10	+ 9	+5	+2	1849 July–Se
6	+5	+ 2	− 5	−12	− 9	− 1	+2	+7	+14	+17	+15	+8	+3	1851 Aug.–Oc
7	0	− 3	− 7	− 8	− 7	− 3	0	+3	+ 4	+ 4	+ 3	+2	−1	1853 June–Ju
8	−1	− 3	− 6	− 8	− 4	+ 2	+4	+5	+ 8	+ 9	+ 7	+1	+1	1854 Sept.
9	−5	−10	−14	−15	−12	−10	−8	−5	− 2	− 1	− 3	−4	−7	1855 May–Ju
10	+3	− 1	− 3	− 4	− 2	− 1	+2	+6	+11	+12	+ 9	+6	+3	1855 Sept.–O
12	+6	+ 4	+ 3	+ 3	+ 3	+ 4	+7	+9	+12	+13	+12	+9	+7	1856 July–Se
13	−4	− 4	− 5	− 6	− 5	− 3	−2	−2	+ 2	+ 5	+ 3	−3	−2	1856 Oct.–De
17	+1	0	− 3	− 4	− 3	− 1	+1	+2	+10	+15	+11	+1	+2·5	1858 Nov.–D
19	−3	− 6	−12	−13	−10	− 4	−1	+1	+ 6	+ 8	+ 6	+1	−2	1860 Apr.–Ju
20	−3	− 6	−11	−13	−11	− 7	−4	−2	0	0	0	−1	−5	1861 Apr.–Ju
23	−1	− 3	− 5	− 6	− 5	− 3	−1	+2	+ 3	+ 4	+ 3	+2	−1	1877 Sept.
Mean	0	− 3	− 6	− 8	− 6	− 3	0	+3	+ 6	+ 8	+ 6	+2	0	
						b. *Examples with Max. and Min. between two points.*								
1	+1	− 1	− 1	0	+ 1	+ 3	+5	+6	+10	+11	+ 8	+5	+4	1841 Sept.–O
5	−3	− 6	− 8	− 8	− 4	+ 1	+1	0	+ 4	+ 7	+ 5	+2	−1	1850 Aug.–Se
11	−6	−13	−14	−14	−11	− 6	+1	+7	+ 9	+ 8	+ 6	+3	−2·5	1856 June–J
14	−6	− 8	− 9	− 9	− 7	− 3	−1	0	+ 4	+ 5	+ 2	−3	−3	1857 Apr.–M
15	+5	+ 3	+ 2	+ 2	+ 3	+ 5	+7	+8	+12	+13	+12	+8	+7	1857 Aug.–Se
16	−5	−12	−15	−14	−10	− 6	−4	−3	− 1	− 1	− 2	−5	−6·5	1858 May–Ju
18	0	− 1	− 3	− 3	− 2	− 1	−1	+5	+10	+ 9	+ 6	+1	+2	1859 May–Ju
Mean	−2	− 5	− 7	− 7	− 4	− 1	+1	+3	+ 7	+ 7	+ 5	+2	0	

The mean amplitude appears to be not quite 8 (= 0·08 mag.): harmonic analysis makes it about 7 in both *a* and *b*, but the form is not quite that of a sine-curve.

The agreement between the two divisions of the table, and the approximation to a symmetrical sine-curve in both, are really remarkable, considering all the conditions.

Nos. 21 and 22 were considered too doubtful to be included, and Nos. 1 and 23 are also formed from very scanty material. It may be of interest to give the details of No. 1 to illustrate this. The figures are for consecutive cycles of 6·454 days. The mean position of such a group as this is obviously rather difficult to determine.

	(1).	(2).	(3).	(4).	(5).	(6).
a	...	+9				
b	+8	−1	+9	...	...	+2
c	−6	+1	...	...	...	+2
d	...	...	+4	+10	...	−6
e						
f	...	...	...	+12		
g	−2					
Mean	0	+3	+6·5	+11	...	−1

The majority are better than this, but all are scrappy enough to make any satisfactory results a pleasing surprise.

No alteration was made in the residuals, interpolations were very sparingly introduced, and there was no "smoothing" except in Nos. 17 and 22.

Summary.

(1) Baxendell's 1220 observations of β Lyræ, extending over thirty-eight years (representing more than 1000 periods, many of which, however, are unobserved) made on different plans, have been reduced to one system.

(2) The formula

$$2398590^{d}\cdot57 + 12^{d}\cdot908E + 0^{d}\cdot0000037E^{2}$$

for date of principal minimum was deduced from them, with a symmetrical light curve shown in Table V.

(3) But there is apparently a small seasonal effect (see Table VI.) and possibly other long-period effects.

(4) And there is a subsidiary variation in about 6·576 days, the maximum and minimum of which show an apparent long-period fluctuation in about 552 × 6·454 days, swinging 68° on either side of the mean value. The data are not sufficient to afford convincing proof of this fluctuation without further corroboration; but of the existence of the subsidiary variation itself there can be no doubt.

On the Photometric Intensity of the Corona at the Eclipse of 1922 *September* 21 *at Wallal.* By A. D. Ross, M.A., D.Sc., F.R.S.E., Professor of Mathematics and Physics in the University of Western Australia, Member of the Crocker Expedition of the Lick Observatory, 1922.

During the past half century many determinations have been made at eclipses of the brightness of the coronal light. Early measurements gave results which are now known to be much too high, and placed the total brightness at anything from 3 to 25 times the light of the full moon. While some recent estimates give a value as high as 3 to 5 times the light of the full moon, the majority of determinations indicate a value rather less than that of the moon. The following are some recent measurements :—

*	1886 eclipse,	Corona = 0·74 full moon.
†	1889 ,,	0·57
‡	,, ,,	0·64
§	1893 ,,	1·31
‖	,, ,,	1·41
¶	1905 ,,	0·75

In view of the excellent atmospheric conditions obtaining at Wallal, the author decided to make a photographic measurement of the total coronal light, and compare it at the same place with that of the full moon.

The photometer used was constructed in the University of Western Australia. A box measuring internally about $6\frac{1}{2} \times 4\frac{1}{4} \times 3\frac{1}{4}$ inches had three draw-slides in its lid running parallel to the length of the box. A wooden block measuring $6\frac{3}{8} \times 1 \times 2\frac{1}{2}$ inches had its $2\frac{1}{2}$-inch thickness pierced with two sets of cylindrical holes, seven in each set, and of diameter about $\frac{1}{2}$ inch. The two $6\frac{3}{8} \times 1$ faces were covered with sheet brass, one face having circular openings 0·15 inch in diameter over each cylindrical hole in the block, and the other having circular openings ranging from 0·04 to 0·4 inch in diameter over the cylindrical holes. The block with its metal facings could be fixed to the inside of the lid immediately below any one of the three draw-slides. All the metal work and woodwork were finished dead black, the circumferences of the holes in the metal plates bevelled to a sharp edge, and devices employed to facilitate the assembling of the apparatus and the light-tight closing of the box. An inner false bottom covered with black velvet was fitted, and supported by springs from the true bottom of the box.

When two quarter-plate photographic plates were laid on the inner bottom, the perforated block attached to the inside of the lid in one of the three positions, and the box closed, the apparatus was ready for use.

* Douglas, *Phil. Trans. Roy. Soc.*, **180**, 363.
† Leuschner, *Phil. Trans. Roy. Soc.*, **187**, 433.
‡ Turner, *Lick Obs.*, "Report on Eclipse of 1889 Jan. 1."
§ Abney and Thorpe, *Phil. Trans. Roy. Soc.*, **187**, 433.
‖ Turner, *Proc. Roy. Soc.*, **66**, 403.
¶ Fabry, *Comptes Rendus*, **141**, 870.

On withdrawing the slide over the perforated block, light passed into the perforations through the two sets of various-sized apertures, and finally fell on the portions of the photographic plates exposed through the 0·15-inch apertures. The magnitude of the chemical effects produced in the emulsion in these parts of the plates would depend on the relative sizes of the apertures through which the light entered the apparatus. To facilitate the subsequent measurements, the areas of the apertures in each set had been arranged to form a geometric series from a circle of diameter 0·04 inch for the first to a circle of diameter 0·4 inch for the seventh. To prevent the formation of pinhole images a screen of finely ground glass was mounted at a short distance above the lid of the photometer box, and the whole was contained in a rectangular tube 4 feet 6 inches long, so that the apparatus could be used to measure the intensity of a light towards which the tube was directed.

By having three draw-slides in the lid of the box it was possible to secure, on the same plate, records of the light intensity of three sources, and the same series of apertures was employed in each test.

For the eclipse tests an Ilford Special Rapid Plate and a Wellington Antiscreen Plate were used simultaneously in the box. On the former were obtained two records of the light of the full moon on 1922 September 6, and one of the coronal light at the eclipse on September 21. On the later plate were recorded the light of the full moon on September 6, the coronal light on September 21, and the light from a Hefner standard lamp burning amyl-acetate. The exposures in the case of the moonlight were made when the moon had the same altitude as the sun in eclipse. In all tests the same exposure of 4 mins. 0 secs. was given. After the three exposures were completed the plates were developed and fixed.

The negatives were afterwards examined in a low-power microscope, an optical device being employed whereby any three of the dusky patches could be viewed simultaneously side by side in beams of light of equal intensity. As the corona was not greatly inferior to the moon in brightness, all the records were useful in the comparisons. The amyl-acetate lamp was also used at a distance of between 7 and 8 feet in the photographic test, and this was found to give records suitable for the purpose.

With the Ilford Special Rapid Plate the mean value of the total intensity of the coronal light was found to be 0·62 as compared with the moonlight of September 6, when the moon was east of the meridian, and 0·59 as compared with the moonlight when the moon was at the same altitude west of the meridian. With the Wellington Antiscreen Plate a mean value of 0·77 times the intensity of the moonlight was obtained. The amyl-acetate lamp test gave 0·18 lux for the brightness of the full moon.

It will be seen that the two different plates give distinctly different values for the ratio of coronal to lunar light, but this is undoubtedly due to the difference in sensitiveness. The Antiscreen Plate is highly sensitive to the green light, and, unlike the Ilford Special Rapid Plate, is even to some extent sensitive to yellow rays. One concludes, therefore, that the intensity of visual radiations in the coronal light bears a higher ratio to those in the light of the full moon than does the actinic radia-

tions. These results, compared with those obtained by previous experimenters, suggest a real variation in the brightness of the corona at different periods—possibly increased brightness at the times of sunspot maxima and reduced brightness at sunspot minima. Moreover, it is probable that the ratio of less refrangible to more refrangible radiation varies from year to year, and it seems desirable that this matter should receive careful investigation. The usual forms of spectro-photometer are hardly suitable for eclipse work, and the simpler expedient of comparing results with the same brands of ordinary and orthochromatic plates in the same photometer would probably give results of sufficient accuracy.

The author desires to express his indebtedness to Mr. W. H. Crocker of San Francisco and to Dr. W. W. Campbell of the Lick Observatory for facilities for carrying out the above tests at Wallal.

University of
Western Australia, Perth:
1924 *April* 26.

On the Conditions for the Observation of the Total Solar Eclipse of 1926 *January* 14. By A. D. Ross, M.A., D.Sc., F.R.S.E., and R. D. Thompson, M.A., M.Sc., A Inst.P.

The conditions for the total eclipse of the sun on 1926 January 14 appear to warrant arrangements for the making of astronomical observations. The belt of totality stretches over a wide tract of ocean, but it also crosses the islands of Sumatra, Banka, Borneo, and Mindanao, and while the maximum duration of totality of 4 min. 11 secs. occurs in the Indian Ocean, the duration is still as great as 3 mins. 21 secs. in western Sumatra. The following table indicates briefly the main features of the total eclipse:—

Station.	Duration.	Time of Mid-eclipse.	Sun's Altitude.
	m s	h m	°
S.W. coast of Sumatra	3 21	7 35·4 G.M.T.	53
(102° 10′ E., 4° 03′ S.)		2 10·1 L.M.T.	
Palembang, S.E. Sumatra	2 59	7 41·4 G.M.T.	49
(104° 44′ E., 3° 01′ S.)		2 40·3 L.M.T.	
W. coast of Banka Island	3 07	7 43·9 G.M.T.	45
(105° 48′ E., 2° 20′ S.)		2 47·1 L.M.T.	

The first station mentioned, on the south west coast of Sumatra, is only about 14 miles from the town of Benkoelen, the capital of the Dutch residency in that part of the island. To this town there is a fast fortnightly service of Dutch ships from Batavia. Palembang, too, is an important town on the river Musi, about 50 miles from its mouth, and is also easily accessible from Batavia and Singapore. The belt of

totality is here about 80 miles wide, and the central line passes only 10 miles from the town. On the central line in this region the duration of totality is about 3 mins. 12 secs. Thereafter the belt of totality crosses Banka Island, which, like its neighbour Billiton Island, is an important mining district. Points farther to the east would be less satisfactory as observing stations, owing to the shorter duration of the total phase and the diminished altitude of the eclipsed sun.

Turning now from the astronomical conditions we come to the all-important matter of weather. All the stations are in the tropics and close to the equator, and accordingly suffer from that general tendency to cloudiness in the afternoon. *Verhandlungen* No. 8, parts 1 to 4, issued by the Koninklijk Magnetisch en Meteorologisch Observatorium te Batavia, contains a large amount of detailed information regarding the climate of the Dutch East Indies. From this work it appears that in the month of January the west monsoon blows with full force over the seas between Borneo and Sumatra, while the upper air-currents are in a different direction and favour the formation of much high stratus and alto-cumulus cloud. The chief manager of the Billiton Company agrees that January and February are bad months for observations in that island. On the west coast of Sumatra the monsoon is practically absent. There the meteorological conditions are much more favourable, and in January light land and sea breezes are the rule. At the hour of the eclipse a south-west wind of velocity about 6 miles per hour is probable. Through the kindness of Dr. C. Braak, of the Batavia Observatory, the authors were able to arrange for a number of special observations to be made in 1924 January, including observations of the amount of cloud in the afternoon hours. Averaging the readings obtained, the amount of cloud between 2 and 3 p.m. (on a scale of 0 clear and 10 overcast) may be taken as—

4 on the west coast of Sumatra.
7 for Banka and Billiton Islands.
4 for Discovery Oostbank.
6 for Pontianak (west coast of Borneo).

Discovery Oostbank is a small reef (109° 10′ E., 3° 35′ S.) to the south-east of Billiton Island, rising only one or two metres above sea-level.

The average rainfall for the entire month of January is about 12 inches at all the places mentioned above, but the rain is very irregular in its incidence, as it comes largely from thunder-clouds.

The observations show generally—

(i) That while at all stations the amount of cloud in the afternoon is greater than in the forenoon, the increase is less at coast than at inland stations.

(ii) That the increase in cloud towards the afternoon is still less marked in the case of small islands such as Discovery Oostbank.

(iii) That the amount of cloud varies greatly from day to day at the same station, and that on the same day the amount of cloud may be altogether different at two stations separated by 50 to 100 miles.

We would therefore suggest the suitability of a point on the central line near Benkoelen as an observing station, and the desirability of a second eclipse party going to the neighbourhood of Palembang. We shall be pleased to place at the disposal of any prospective parties any further information available as to the various localities.

In conclusion, we would express our gratitude to Dr. Braak for the readiness with which he collected the special data which we considered necessary as a basis for this report.

University of Western Australia,
Perth: 1924 *May* 3.

Occultation of Regulus. By W. E. Cooke.

The Occultation of Regulus was observed at the Sydney Observatory on 1924 April 15.

Instrument 11¼-inch equatorial. Observer H. E. Rayner.

	d	h	m	s	
Immersion	April 14,	19	03	49·8	G.M.T.
Emersion		20	13	23·8	